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# Anatomy of Top-Mode Extended Technicolor Model <sup>\*</sup>

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## Abstract

We analyze two versions of the extended technicolor (ETC) incorporating the top quark condensate via the flavor-universal coloron type topcolor  $SU(3)_1 \times SU(3)_2$ : A straightforward top-mode ETC having quarks and techniquarks assigned to a single (strong)  $SU(3)_1$ , and a “twisted model” with techniquarks carrying the weak  $SU(3)_2$  while quarks the strong  $SU(3)_1$ . The straightforward model has the same ETC structure as that of Appelquist et al. without topcolor which we first analyze to find that it yields only too small ETC-induced mass for the third generation. In contrast, our model having topcolor takes the form of a version of the topcolor-assisted technicolor (TC2) after ETC breakings, which triggers the top quark condensate giving rise to a realistic top mass. However, techniquarks have the strong topcolor  $SU(3)_1$  in addition to the already strong walking/conformal technicolor, which triggers the techniquark condensate at scale much higher than the weak scale, a disaster. We then consider a “twisted model” of TC2, though not an explicit ETC. We find a new feature that “ETC”-induced quark mass is enhanced to the realistic value by the large anomalous dimension  $\gamma_m \simeq 2$  of Nambu-Jona-Lasinio-type topcolor interactions. The result roughly reproduces the realistic quark masses. We further find a novel effect of the above large anomalous dimension  $\gamma_m \simeq 2$ : The top-pion mass has a universal upper bound,  $m_{\pi_t} < 70 \text{ GeV}$ , in the generic TC2 model.

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# 1 Introduction

The Standard model (SM) has a mysterious part, the electroweak symmetry breaking (EWSB), to give masses of quarks, leptons and W/Z-bosons. The EWSB via the elementary Higgs field in the SM has some problems. In order to solve these problems, we should build a scenario beyond the SM. One of candidates for such scenarios is Technicolor (TC) [1]. TC is an attractive idea for the EWSB without the elementary Higgs, based on an analogy to the QCD with technifermion condensate, instead of the quark condensate in QCD, responsible for the mass of W/Z bosons. In order to give mass to the SM fermions, we have to extend the TC into a larger picture which communicates the technifermion to the SM fermion. A typical one is the extended TC (ETC) model [2] in which the TC group is embedded into a larger gauge group including the horizontal gauge group of three families of the SM fermion.<sup>1</sup> This old-type ETC model which is based on the scale-up of the QCD has some problems; Flavor Changing Neutral Current (FCNC) problem [2], light pseudo Nambu-Goldstone (NG) bosons and deviation from the LEP precision experiments, the so-called  $S$  parameter problem [4].

The FCNC problem and light pseudo NG bosons have already been solved by the walking (=scale invariant/conformal) TC [5, 6] with the TC gauge coupling almost constant or running very slowly, which was shown to develop a large anomalous dimension  $\gamma_m \simeq 1$  [6]. (For reviews see Ref. [7, 8, 9].) It was suggested [10, 11] that a realistic walking/conformal gauge theory is given by the large  $N_f$  QCD, a jargon for a version of the “QCD” with  $N_c$  colors and many massless flavors  $N_f (\gg N_c)$ , in which the two-loop beta function possesses the Banks-Zaks infrared fixed point (BZ-IRFP)  $\alpha_*$  for large  $N_f$  [12].<sup>2</sup> such that  $N_f^* < N_f < 11N_c/2$ , with  $N_f^* \simeq 8.01$  for  $N_c = 3$ . Looking at the region  $0 < \alpha < \alpha_*$ , we note that  $\alpha_* \searrow 0$  when  $N_f \nearrow 11N_c/2$ , and hence there exists a certain region ( $N_f^* < N_f^{\text{cr}} < N_f < 11N_c/2$ ) (“conformal window”) such that  $\alpha_* < \alpha^{\text{crit}}$ , where  $\alpha^{\text{crit}}$  is the critical coupling for the chiral symmetry breaking and hence the chiral symmetry gets restored  $\langle \bar{\psi}\psi \rangle = 0$  in this region. Here  $\alpha^{\text{crit}}$  may be evaluated as  $\alpha^{\text{crit}} = \pi/3C_2(F)$  in the ladder approximation, in which case we have  $N_f^{\text{cr}} \simeq 4N_c$  [10].<sup>3</sup> When applied to TC, we set  $\alpha_*$  slightly larger than  $\alpha^{\text{crit}}$  (slightly outside of the conformal window), with the running coupling becoming slightly larger than the critical coupling in the infrared region, we have a condensate or the dynamical mass of the technifermion  $m_{\text{TC}}$  which is much smaller than the intrinsic scale of the theory  $\Lambda_{\text{TC}} (\gg m_{\text{TC}})$ . In a wide region  $m_{\text{TC}} < \mu < \Lambda_{\text{TC}}$  the coupling is walking due

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<sup>1</sup> Another possibility is a composite model where both the SM fermions and the technifermions are composite on equal footing, in which case the residual four-fermion interactions among composites play the role of the ETC-induced effective four-fermion interactions between technifermions and SM fermions. [3]

<sup>2</sup> There is another possibility to have the Banks-Zaks IR fixed point without large  $N_f$  by introducing higher dimensional representation [13].

<sup>3</sup> In the case of  $N_c = 3$ , this value  $N_f^{\text{cr}} \simeq 4N_c = 12$  is somewhat different from the lattice value [14],  $6 < N_f^{\text{cr}} < 7$ .

to the BZ-IRFP and the theory develops a large anomalous dimension  $\gamma_m \simeq 1$  and enhanced condensate  $\langle \bar{\psi}\psi \rangle|_{\Lambda_{\text{TC}}} \sim \Lambda_{\text{TC}} m_{\text{TC}}^2$  at the scale of  $\Lambda_{\text{TC}}$  which is identified with the ETC scale  $\Lambda_{\text{TC}} = \Lambda_{\text{ETC}}$ .

As to the  $S$  parameter, it is rather difficult at this moment to draw a definite conclusion in the walking/conformal TC, since there is no reliable non-perturbative calculation of the  $S$  parameter in the walking/conformal TC which is strongly coupled and has no simple scale-up of the known dynamics like QCD. Actually, it was argued [15] that the  $S$  parameter is suppressed in walking/conformal theories with  $\gamma_m \simeq 1$ . Straightforward computation of the  $S$  parameter was also performed, based on the ladder Schwinger-Dyson (SD) equation and Bethe-Salpeter equation, which indicates decreasing tendency in the walking/conformal regime [16]. Recently, a reduction of the  $S$  parameter was further claimed [17] in a version of the holographic QCD with deformation of the replacement of the anomalous dimension  $\gamma_m \simeq 0$  by  $\gamma_m \simeq 1$  based on AdS/CFT correspondence.

Therefore, it would be worth engineering an ETC model building based on the large  $N_f$  QCD near the conformal window as a walking/conformal TC. There have been such an attempt [18, 19] based on a generalized version of Most Attractive Channel (MAC) analysis, which claimed reasonable phenomenological result. However, it does not seem to explain a large mass of the top quark (top-bottom splitting) in a way consistent with the  $\rho/T$  parameter constraint from the LEP precision experiments, the problem being more serious for the walking/conformal TC. [20]

Such a large top quark mass may be explained by the top quark condensate, or top-mode standard model [21, 22, 23, 24]. So, it would be natural to seek a model which accommodates top quark condensate into an explicit scheme of ETC. After ETC breaking, it would yield a low energy effective theory something resembling the topcolor-assisted technicolor (TC2) [25, 26] as a variant of the flavor-universal TC2 [27] rather than of others: “classic TC2” [25] and “type II TC2” [26].

In this paper we experiment such an explicit ETC model incorporating straightforwardly the flavor-universal coloron type topcolor. Apart from the topcolor sector, the model is the same as that of Ref. [18, 19]; Namely, the ETC gauge group  $SU(5)_{\text{ETC}}$  contains one-family  $SU(2)_{\text{TC}}$  walking/conformal technicolor and  $SU(3)$  horizontal gauge symmetry for three families of the SM fermions. Since the flavor-universal coloron acts like  $SU(3)_{\text{QCD}}$  gluon on the quarks and techniquarks, the model is the same as that of Ref. [18, 19] concerning the ETC sector. It was shown in Ref. [18, 19] that  $SU(5)_{\text{ETC}}$  can break down successively to  $SU(4)_{\text{ETC}}$ ,  $SU(3)_{\text{ETC}}$  and eventually to  $SU(2)_{\text{TC}}$ , the walking/conformal TC, by the Most Attractive Channel (MAC) analyses of the dynamics of ETC and  $SU(2)_{\text{HC}}$  hypercolor. However, actual criticality conditions were not fully considered for the ETC breakings and all three ETC breaking scales  $\Lambda_1 > \Lambda_2 > \Lambda_3$  were thus treated as free parameters. Here we impose criticality condition for every step of the ETC breakings, based on the ladder SD equation, and find that the ETC breaking scales are no longer free parameters but actually are determined once we fix the scale  $\Lambda_1$  for the initial breaking  $SU(5)_{\text{ETC}} \rightarrow SU(4)_{\text{ETC}}$  as an input. It turns out

that the scales for the second and third stages of the ETC breakings,  $\Lambda_2, \Lambda_3$ , are much higher than in Ref. [18, 19] and hence yield extremely small mass  $\mathcal{O}(10^{-1} \text{ GeV})$  for the third generation fermions characterized by  $\Lambda_3$ , even though the TC condensate is enhanced by the large anomalous dimension of the walking/conformal TC.

Then we consider a straightforward extension of such an ETC model (“top-mode ETC”) so as to incorporate the flavor-universal  $SU(3)_1 \times SU(3)_2$  topcolor, by simply replacing the  $SU(3)_{\text{QCD}}$  of both techniquarks and quarks in the conventional ETC model [18, 19] by a single factor group  $SU(3)_1$  of the topcolor. The topcolor breaks down to  $SU(3)_{\text{QCD}}$  color symmetry, giving rise to the coloron mass  $M_C$  and yields effective four-fermion interaction among quarks which is tuned close to the critical coupling. This is a new ingredient absent in the conventional ETC model [18, 19]. Then the broken ETC-induced four-fermion interactions prefer the third generation quark condensate and the  $U(1)_Y$  interaction further prefers the top quark condensate to the bottom condensate. Accordingly, the top quark condensate takes place and the top quark acquires the mass  $\hat{m}_t$  from the top quark condensate which is expected to dominate the ETC-induced mass  $m_t^{(0)}$  ( $\ll \hat{m}_t$ ):  $m_t = \hat{m}_t + m_t^{(0)}$ . Thus the model appears to solve the third generation mass problem of the ETC model of Ref. [18, 19].

However, since we require the topcolor  $SU(3)_1$  is near criticality and thus is very strong, the techniquarks, carrying the same strong topcolor as well as the equally strong walking/conformal technicolor, are forced to condense at the scale much larger than the weak scale, a disaster.

Then we consider an alternative (“twisted model”), namely a version of TC2 with the flavor-universal coloron type topcolor  $SU(3)_1$  for the quarks but  $SU(3)_2$  for the techniquarks. Although explicit ETC model building of this type of TC2 is rather complicated and not available at this moment, we hope that some larger picture would make it. Once we admit a possibility that such an “ETC” gives rise to hierarchical scales to discriminate the generations of quarks/leptons as in the ordinary ETC, we can reproduce the quark/lepton masses including the third generation: Only the top quark has mass from the top quark condensate as well as from the TC condensate via “ETC”-induced four-fermions, while other fermions acquire mass only from the latter.

A novel feature we find is the large enhancement of the “ETC”-induced mass which acts like the bare mass explicitly breaking the chiral symmetry of the topcolor sector. Here we recall the fact [28, 29] that the Nambu-Jona-Lasinio (NJL) -type four-fermion interactions develop large anomalous dimension  $\gamma_m \simeq 2$  in *both broken and symmetric phases* as far as the couplings are close to the criticality<sup>4</sup>. This implies great amplification of the bare mass  $m^{(0)}$  defined at the cutoff scale  $\Lambda$  to the renormalized mass  $m(\mu)$

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<sup>4</sup> Although the pure NJL in four dimensions is not renormalizable and the concept of the anomalous dimension is not well-defined, the present case of the broken topcolor with QCD coupling is actually the gauged NJL model which is renormalizable in both broken and symmetric phases when  $A = 24/(33 - 2N_f) > 1$  [29, 30] and in fact in our case  $A = 8/7 > 1$  for  $N_f = 6$  ( $\mu < m_{\text{TC}}$ ) and  $A = 24/13 > 1$  for  $N_f = 10$  ( $m_{\text{TC}} < \mu < M_C$ ). In this case the anomalous dimension has a log correction to 2:  $Z_m^{-1} \simeq (\Lambda/\mu)^2 [\ln(\Lambda/\Lambda_{\text{QCD}})/\ln(\mu/\Lambda_{\text{QCD}})]^{-A/2}$ . See Ref.[8].

defined at the lower scale  $\mu (\ll \Lambda)$  (usually the dynamical mass of the fermion):  $m(\mu) = Z_m^{-1} m^{(0)}$  with  $Z_m^{-1} = (\Lambda/\mu)^{\gamma_m} \simeq (\Lambda/\mu)^2$ . In the case at hand, while the unbroken topcolor develops rather small anomalous dimension for the region  $M_C < \mu < \Lambda_3$ , the broken topcolor for the region  $\hat{m}_t (\simeq m_t) < \mu < M_C$  yields a large anomalous dimension to the top and bottom quarks whose four-fermion couplings (in the broken phase for top, while in the symmetric phase for bottom) are both close to the criticality. This amplifies the bare mass by the factor  $Z_m^{-1} \simeq (M_C/m_t)^2 [\ln(M_C/\Lambda_{\text{QCD}})/\ln(m_t/\Lambda_{\text{QCD}})]^{-A/2}$  (with log correction due to QCD) which would be  $Z_m^{-1} > 500$  for e.g.,  $M_C > 4 \text{ TeV}$ . This easily realizes renormalized ETC-induced mass  $\simeq 5 \text{ GeV}$  for the top and bottom. This gives us the renormalized ETC-induced mass for the top and bottom even when the typical ETC-induced bare mass at the scale of ETC breaking is very tiny:  $\sim \mathcal{O}(100 \text{ MeV})$ . Then the main portion of the top mass coming from the top quark condensate is  $\hat{m}_t = m_t - 5 \text{ GeV} = 167 \text{ GeV}$  so that we can reproduce the physical top mass:  $m_t \simeq 167 + 5 = 172 \text{ GeV}$ . Other quarks having four-fermion couplings somewhat smaller than the criticality will have the anomalous dimension smaller than that of the third generation, so that the enhancement would be much smaller.

So we hope that it can be the first step toward constructing a realistic model incorporating both technicolor and topcolor.

As a novel effect of the large anomalous dimension  $\gamma_m \simeq 2$  of the topcolor dynamics mentioned above, we find the upper bound of the top-pion mass  $m_{\pi_t} < 70 \text{ GeV}$  which is conservative estimate and is universal to the generic TC2 not restricted to our model setting. The top-pion appears in the generic TC2 model[25, 26], since both the top quark condensate and technifermion condensate break respective global symmetries, which results in two kinds of three Nambu-Goldstone (NG) bosons. Three of them (mainly boundstates of technifermions) are absorbed into  $W, Z$  bosons as usual and the rest three are pseudo NG bosons (mainly boundstates of top and bottom). The top-pion mass may be estimated by the Dashen formula [31]  $m_{\pi_t}^2 f_{\pi_t}^2 = m_t^{(0)} \langle \bar{t}t \rangle$ , where  $m_t^{(0)}$  is the ETC-induced top mass acting as the bare mass. The decay constant  $f_{\pi_t}$  may be calculated by the Pagels-Stokar formula [32] for  $f_{\pi_t}^2$  which is evaluated in exactly the same way as in the original top quark condensate paper [21] where the mass function is not a constant but logarithmically damping due to the QCD correction. It reads:  $f_{\pi_t}^2 = [N_c(\hat{m}_t)^2/(8\pi^2)] \cdot F(M_C, \hat{m}_t)$ , where the function  $F$  is  $\ln(M_C^2/(\hat{m}_t)^2)$  in the pure NJL model but is modified to a certain function in the gauged NJL model which is finite in the limit  $M_C \rightarrow \infty$ , reflecting the renormalizability of the gauged NJL model.

Now, the crucial point of our estimate is that the right-hand side of the Dashen formula is *renormalization-point independent*:  $m_t^{(0)}(M_C) \cdot \langle \bar{t}t \rangle|_{M_C} = m_t^{(0)}(\hat{m}_t) \cdot \langle \bar{t}t \rangle|_{\hat{m}_t}$ . Using the estimation  $\langle \bar{t}t \rangle|_{\hat{m}_t} = N_c(\hat{m}_t)^3/(4\pi^2)$ , we have  $m_{\pi_t}^2 = m_t^{(0)} \langle \bar{t}t \rangle / f_{\pi_t}^2 = 2m_t^{(0)}(\hat{m}_t) \cdot \hat{m}_t / F(M_C, \hat{m}_t) = 2x(m_t - x) / F(M_C, \hat{m}_t = m_t - x)$ , where we have written  $x \equiv m_t^{(0)}(\hat{m}_t)$ . The gross structure of this expression is essentially determined by the factor  $2x(m_t - x)$  which has a maximum value  $m_t^2/2$  at  $x = m_t/2$ , and the function  $F(M_C, \hat{m}_t = m_t - x)$ , numerically similar to  $\ln(M_C^2/(m_t - x)^2)$ , is a slowly increasing function as  $M_C$  (not

diverging, though) and hence the lowest possible  $M_C$  gives the upper bound of  $m_{\pi_t}^2$ . A model-independent lower limit of the mass of the flavor-universal coloron (as in the class of models considered in this paper)  $M_C$  is  $M_C > 837 \text{ GeV}$  [33], which yields an upper bound  $m_{\pi_t}^2 < (60 \text{ GeV})^2$  at  $x \simeq m_t/2$ . On the other hand, flavor-non-universal coloron mass bound is somewhat weaker  $M_C > 450 \text{ GeV}$  which implies the upper bound  $m_{\pi_t}^2 < (70 \text{ GeV})^2$ . The latter is a very conservative upper bound universal to generic model of TC2 not restricted to specific TC2 model, since in the generic TC2 model the actual mass bound of the coloron is  $M_C/\cot\theta > 837 \text{ GeV}$  and  $M_C/\cot\theta > 450 \text{ GeV}$  for flavor-universal and flavor-non-universal cases, respectively, with the condition that  $\cot\theta > 4$  in order to trigger the top quark condensate [33].

The paper is organized as follows: In Sec. 2, we present an ETC model incorporating the topcolor (Top-Mode ETC). The successive ETC breakings through MAC analyses is shown in the same way as the model without topcolor. In Sec. 3, we discuss criticality conditions for the ETC breakings and estimate the breaking scales, based on the ladder SD equation analysis. We find that without topcolor the model yields only a small mass on the order of  $\mathcal{O}(0.1 \text{ GeV})$  to the third generation. In Sec. 4, we study the effect of the topcolor in the form of the gauged NJL model, the effective four-fermion interactions due to the broken ETC and topcolor interactions with the SM gauge interactions. Based on the phase structure of the gauged NJL model, we discuss the criticality conditions under which top quark is the only quark to condense. Then it is shown that the techniquark condensate, which is triggered by the combined effects of the technicolor and the topcolor, is very large as far as we require the topcolor coupling is large enough to trigger the top quark condensate. In Sec. 5 we argue an alternative model of TC2, where we roughly estimate third generation quark masses and emphasize that the large anomalous dimension  $\gamma_m \simeq 2$  of the broken topcolor as well as the large anomalous dimension of the walking/conformal TC is crucial to give the large enhancement for the third generation quark masses. In Sec. 6 we give a new estimate of the mass of top-pion as a novel effect of the large anomalous dimension of the broken topcolor. We find a rather small upper bound of the top-pion, which is universal to the generic TC model. Sec. 7 is devoted to summary and discussions.

## 2 A Top Mode Extended Technicolor Model

### 2.1 The Model

We use a typical one-family TC model [9] with  $N_f = 8$  technifermions and with  $SU(2)_{\text{TC}}$  as a TC gauge group, which is a walking/conformal TC near the edge of the conformal window  $N_f \sim 4N_c$  [10] evaluated in the ladder approximation. The simplest ETC model would be  $SU(5)_{\text{ETC}}$  which accommodates  $SU(2)_{\text{TC}}$  one-family technifermions and three families of quarks and leptons. Following Ref. [19], we introduce  $SU(2)_{\text{HC}}$  in order that the  $SU(5)_{\text{ETC}}$  successively breaks down as  $SU(5)_{\text{ETC}} \rightarrow SU(4)_{\text{ETC}} \rightarrow SU(3)_{\text{ETC}} \rightarrow SU(2)_{\text{TC}}$ . Although this model is the same as that of

field	$SU(5)_{\text{ETC}}$	$SU(3)_1$	$SU(3)_2$	$SU(2)_L$	$U(1)_Y$	$SU(2)_{\text{HC}}$
$\mathcal{Q}_L$	5	3	1	2	1/6	1
$\mathcal{U}_R$	5	3	1	1	2/3	1
$\mathcal{D}_R$	5	3	1	1	-1/3	1
$\mathcal{L}_L$	5	1	1	2	-1/2	1
$\mathcal{E}_R$	5	1	1	1	-1	1
$\psi_R$	10	1	1	1	0	2
$\psi'_R$	$\overline{10}$	1	1	1	0	1
$\omega_R$	1	1	1	1	0	2
$\Phi$	1	3	$\overline{3}$	1	0	1

Table 1: Particle contents. This table without  $\Phi$  is the same as Ref. [19] beside that  $SU(3)_{\text{QCD}}$  is replaced by  $SU(3)_1 \times SU(3)_2$ .  $\Phi$  is the scalar field and other fields are the fermion fields.

Ref. [19] as to the ETC sector, we introduce the universal coloron-type topcolor symmetry  $SU(3)_1 \times SU(3)_2$  which breaks down to  $SU(3)_{\text{QCD}}$ . Note that the universal coloron does not affect the ETC sector.

Particle contents in this model is listed in Table. 1.  $\mathcal{Q}, \mathcal{U}, \mathcal{D}, \mathcal{L}, \mathcal{E}$  include one-family technifermions and three-generation quarks and leptons :

$$\mathcal{Q}_L = (Q^a, q_3, q_2, q_1)_L^T, \mathcal{L}_L = (L^a, l_3, l_2, l_1)_L^T, \mathcal{U}_R = (U^a, t, c, u)_R^T, \dots, \quad (1)$$

where  $a = 1, 2$  is TC indices,  $q_i(l_i)$  represents  $i$ -th generation  $SU(2)_L$ -doublet quark(lepton),  $Q^a = (U^a, D^a)^T$  and  $L^a = (N^a, E^a)^T$  are techniquarks and technileptons, respectively. Additional fermions  $\psi_R, \psi'_R$  participate in the desirable ETC breaking as will be discussed in detail in Sec. 2.2, while  $\omega_R$  contributes only to the running behavior of the  $SU(2)_{\text{HC}}$  gauge coupling, and the largest possible number of  $\omega$  is  $N_\omega \leq 10$  in order to keep the asymptotic freedom ( $N_\omega = 2$  in Ref. [19], while we shall take  $N_\omega = 10$ ). The condensation of “effective Higgs” field  $\Phi$  breaks the topcolor symmetry to  $SU(3)_{\text{QCD}}$  symmetry.

Apart from the ETC group, this is the same as the flavor-universal TC2 [27] in the sense that all quarks (techniquarks as well) have the same charge under the topcolor symmetry  $SU(3)_1 \times SU(3)_2$ , but for simplicity we do not introduce the additional (strong)  $U(1)'$  which in the flavor-universal TC2 models is coupled only to the third generation quark to trigger the top condensate. Instead, the ETC gauge interaction in our case discriminates the third generation quarks from others near the criticality of the strong coloron interaction. Once the third generation is selected, top is distinguished from bottom by the usual  $U(1)_Y$  interaction in the Standard Model near the criticality.

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<sup>5</sup> This requires some fine tuning which may be avoided by introducing extra strong  $U(1)'$  as in the

Before discussing detailed dynamics, we here note that the desired ETC breaking:  $SU(5)_{\text{ETC}} \rightarrow SU(4)_{\text{ETC}} \rightarrow SU(3)_{\text{ETC}} \rightarrow SU(2)_{\text{TC}}$  and topcolor breaking:  $SU(3)_1 \times SU(3)_2 \rightarrow SU(3)_{\text{QCD}}$  can be realized through the (electroweak singlet) “effective Higgs” fields

$$\begin{aligned} H_1^{\text{ETC}} &\sim (5, 1, 1) \text{ (under } SU(5)_{\text{ETC}} \times SU(3)_1 \times SU(3)_2), \\ H_2^{\text{ETC}} &\sim (\bar{4}, 1, 1) \text{ (under } SU(4)_{\text{ETC}} \times SU(3)_1 \times SU(3)_2), \\ H_3^{\text{ETC}} &\sim (\bar{3}, 1, 1) \text{ (under } SU(3)_{\text{ETC}} \times SU(3)_1 \times SU(3)_2), \\ \Phi &\sim (1, 3, \bar{3}) \text{ (under } SU(2)_{\text{TC}} \times SU(3)_1 \times SU(3)_2). \end{aligned} \quad (2)$$

Here “effective Higgs” fields  $H_1^{\text{ETC}}, H_2^{\text{ETC}}, H_3^{\text{ETC}}$  break  $SU(5)_{\text{ETC}}, SU(4)_{\text{ETC}}$  and  $SU(3)_{\text{ETC}}$  successively through hierarchical condensates  $\Lambda_1 \gg \Lambda_2 \gg \Lambda_3$  where  $\Lambda_i = \langle H_i^{\text{ETC}} \rangle$ .  $\Phi$  breaks  $SU(3)_1 \times SU(3)_2$  through  $\langle \Phi \rangle$  which we take  $\Lambda_3 > \Lambda_C = \langle \Phi \rangle$ .

Although a scenario of dynamically producing such “effective Higgs” fields for the successive ETC breakings was given in Ref [18, 19], it was left unclear whether or not the criticality conditions for each step of ETC breakings can really be met. Here, we shall explicitly examine the criticality conditions and find that in order for the  $SU(5)_{\text{ETC}} \rightarrow SU(4)_{\text{ETC}}$  breaking takes place, the  $SU(5)_{\text{ETC}}$  coupling at the scale of  $\Lambda_1 (= 1000 \text{ TeV})$  should be much larger than that of Ref [18]. Once  $SU(5)_{\text{ETC}}$  breaking takes place due to the coupling this large the desired successive breakings  $SU(5)_{\text{ETC}} \rightarrow SU(4)_{\text{ETC}} \rightarrow SU(3)_{\text{ETC}} \rightarrow SU(2)_{\text{TC}}$  are actually realized. However, we shall demonstrate that the lowest ETC breaking scale  $\Lambda_3$  is determined to be very large;  $\Lambda_3 \simeq 700 \text{ TeV}$  ( $N_\omega = 2$ ) and  $\Lambda_3 \simeq 360 \text{ TeV}$  ( $N_\omega = 10$ ). Even including ambiguity of critical value of the ladder SD equation up to 30% for the  $SU(5)_{\text{ETC}}$ , it can only be  $\Lambda_3 \gtrsim 150 \text{ TeV}$ , so that the third generation mass should be at most in order of  $\mathcal{O}(10^{-1} \text{ GeV})$  even in the walking/conformal TC with  $\gamma_m \simeq 1$  within the framework of only ETC without topcolor.

Now to a model with topcolor. Due to  $\langle \Phi \rangle = \Lambda_C$  the topcolor symmetry  $SU(3)_1 \times SU(3)_2$  is spontaneously broken down to  $SU(3)_{\text{QCD}}$ . We are left with strongly coupled effective four fermion interaction which, combined with broken ETC and  $U(1)_Y$  gauge interactions near the criticality, triggers the top quark condensate giving rise to the main part of the top quark mass  $m_t^{\text{topC}} \simeq 170 \text{ GeV}$ . In the case of  $N_\omega = 10$ , we have  $\Lambda_2 \simeq 850 \text{ TeV}$ ,  $\Lambda_3 \simeq 360 \text{ TeV}$  for  $\Lambda_1 = 1000 \text{ TeV}$ . If the TC dynamics is near critical dynamics then the ETC driven mass of the third generation  $m_{t,b}^{\text{ETC}} \simeq 10^{-1} \text{ GeV}$  which is regarded as the bare mass at  $\Lambda_3$  in the topcolor dynamics and is expected to be amplified to  $m_{t,b}^{\text{ETC}} \simeq 5 \text{ GeV}$  at a scale of top mass by the anomalous dimension of NJL-type  $\gamma_m \simeq 2$  for the quark bilinear operator due to broken topcolor dynamics. Then we would have  $m_t = m_t^{\text{topC}} + m_t^{\text{ETC}} \simeq 175 \text{ GeV}$  and  $m_b = m_b^{\text{ETC}} \simeq 5 \text{ GeV}$ . However there is a serious drawback in this model: Combined effects of the technicolor

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flavor-universal TC2 models. Although the conventional  $U(1)'$  to distinguish the third generation from others may not straightforwardly be incorporated into the ETC model, the flavor-universal  $U(1)'$  [34] is an interesting possibility. We shall study such a case in Sec.5.



and topcolor are very strong, which trigger the techniquark condensate at much larger scale than the weak scale. Then the  $f_{\pi_T}$  does not satisfy the basic requirement of the model setting  $4f_{\pi_T}^2 \simeq 4(110 \text{ GeV})^2 = (246 \text{ GeV})^2 - f_{\pi_t}^2$ ,  $f_{\pi_t}^2 \simeq (100 \text{ GeV})^2$ . We shall discuss a possible way out in the latter section.

## 2.2 MAC analysis of Successive ETC breakings

In this section we review the MAC analysis of  $SU(5)_{\text{ETC}}$  with  $SU(2)_{\text{HC}}$  following Ref. [18, 19], postponing our own discussions on the criticality of the MAC binding strength to Sec.3<sup>6</sup>.

The successive ETC breaking may be realized by  $\psi_R, \psi'_R$  (topcolor and EW singlets) in Table. 1 and the  $SU(2)_{\text{HC}}$  gauge interaction. Let us define  $\Delta C_2(r_1 \times r_2 \rightarrow r_3) \equiv C_2(r_1) + C_2(r_2) - C_2(r_3)$  where  $C_2(r)$  is a quadratic Casimir operator with representation  $r$  under each gauge group. Also,  $g_{N(\text{ETC})}$  is  $SU(N)_{\text{ETC}}$  gauge coupling,  $\alpha_{N(\text{ETC})} = g_{N(\text{ETC})}^2/4\pi$ , and  $g_{2(\text{HC})}$  is  $SU(2)_{\text{HC}}$  gauge coupling,  $\alpha_{2(\text{HC})} = g_{2(\text{HC})}^2/4\pi$ .

### 2.2.1 Realization of $SU(5)_{\text{ETC}}$ breaking down to $SU(4)_{\text{ETC}}$

First,  $\psi_R, \psi'_R$  in Table. 1 take part in  $SU(5)_{\text{ETC}}$  breaking system. The preserving  $SU(2)_{\text{HC}}$  candidates of condensation are :

$$\begin{aligned} (\overline{10}, 1, 1, 1, 1)_0 \times (\overline{10}, 1, 1, 1, 1)_0 &\rightarrow (5, 1, 1, 1, 1)_0, \\ &: k_5^{(5,1)} = \frac{24}{5} \alpha_{5(\text{ETC})}(\Lambda_1), \end{aligned} \quad (3)$$

$$\begin{aligned} (10, 1, 1, 1, 2)_0 \times (10, 1, 1, 1, 2)_0 &\rightarrow (\overline{5}, 1, 1, 1, 1)_0, \\ &: k_5^{(\overline{5},1)} = \frac{24}{5} \alpha_{5(\text{ETC})}(\Lambda_1) + \frac{3}{2} \alpha_{2(\text{HC})}(\Lambda_1), \end{aligned} \quad (4)$$

$$\begin{aligned} (10, 1, 1, 1, 2)_0 \times (10, 1, 1, 1, 2)_0 &\rightarrow (\overline{5}, 1, 1, 1, 3)_0, \\ &: k_5^{(\overline{5},3)} = \frac{24}{5} \alpha_{5(\text{ETC})}(\Lambda_1) - 2 \alpha_{2(\text{HC})}(\Lambda_1), \end{aligned} \quad (5)$$

where representations in the parentheses correspond to  $(SU(5)_{\text{ETC}}, SU(3)_1, SU(3)_2, SU(2)_L, SU(2)_{\text{HC}})_{U(1)_Y}$ , and each  $\Delta C_2$  is  $\Delta C_2(\overline{10} \times \overline{10} \rightarrow 5) = \Delta C_2(10 \times 10 \rightarrow \overline{5}) = 24/5$  for  $SU(5)_{\text{ETC}}$  and  $\Delta C_2(2 \times 2 \rightarrow 1) = 3/2$ ,  $\Delta C_2(2 \times 2 \rightarrow 3) = -2$  for  $SU(2)_{\text{HC}}$ . Here  $\kappa_N^{(A,B)}$  represents a binding strength for each channel labeled by the representations  $(A, B)$  of condensation under  $(SU(N)_{\text{ETC}}, SU(2)_{\text{HC}})$ . We can see easily  $k_5^{(\overline{5},1)} > k_5^{(5,1)} > k_5^{(\overline{5},3)}$ , so that MAC appears to be Eq.(4) rather than Eq.(3) and (5). However, the channel Eq.(4) is forbidden by the Fermi statistics, and hence Eq.(3) is the MAC for the breaking as  $SU(5)_{\text{ETC}} \rightarrow SU(4)_{\text{ETC}}$ . The condensation of this channel corresponds to  $\langle H_1^{\text{ETC}} \rangle = \Lambda_1 \neq 0$  in Sec. 2.1. Once  $k_5^{(5,1)}$  exceeds the

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<sup>6</sup> Since the universal coloron type topcolor acts in the same way as the  $SU(3)_{\text{QCD}}$  in the discussions of ETC breakings, discussions in Sec.2.2 and Sec.3 apply to both the model of Ref [18, 19] and ours.

critical binding strength  $k_{\text{crit}}$  (as will be discussed in Sec. 3),  $SU(5)_{\text{ETC}}$  breaks down to  $SU(4)_{\text{ETC}}$ , and as a result quark/lepton sector is divided below  $\Lambda_1$  as

$$\mathcal{Q}_L \rightarrow \begin{cases} (1, 3, 1, 2, 1)_{1/6} : q_{1L} = (u, d)_L \\ (4, 3, 1, 2, 1)_{1/6} : \mathcal{Q}'_L \end{cases}, \quad \mathcal{L}_L \rightarrow \begin{cases} (1, 1, 1, 1, 1)_{-1/2} : l_{1L} = (\nu_e, e)_L \\ (4, 1, 1, 1, 1)_{-1/2} : \mathcal{L}'_L \end{cases} \quad (6)$$

$$\mathcal{U}_R \rightarrow \begin{cases} (1, 3, 1, 1, 1)_{2/3} : u_R \\ (4, 3, 1, 1, 1)_{2/3} : \mathcal{U}'_R \end{cases}, \quad (7)$$

$$\mathcal{D}_R \rightarrow \begin{cases} (1, 3, 1, 1, 1)_{-1/3} : d_R \\ (4, 3, 1, 1, 1)_{-1/3} : \mathcal{D}'_R \end{cases}, \quad \mathcal{E}_R \rightarrow \begin{cases} (1, 1, 1, 1, 1)_{-1} : e_R \\ (4, 1, 1, 1, 1)_{-1} : \mathcal{E}'_R \end{cases} \quad (8)$$

and the remaining fields of  $\psi_R, \psi'_R$  are

$$\psi_R : \{(4, 1, 1, 1, 2)_0, (6, 1, 1, 1, 2)_0\}, \quad \psi'_R : (\bar{4}, 1, 1, 1, 1)_0, \quad (9)$$

where we labeled the representations according to  $(SU(4)_{\text{ETC}}, SU(3)_1, SU(3)_2, SU(2)_L, SU(2)_{\text{HC}})_{U(1)_Y}$ .

### 2.2.2 Realization of $SU(4)_{\text{ETC}}$ breaking down to $SU(3)_{\text{ETC}}$

Now,  $(4, 1, 1, 1, 2)_0, (6, 1, 1, 1, 2)_0, (\bar{4}, 1, 1, 1, 1)_0$  and the  $SU(2)_{\text{HC}}$  gauge interaction take part in the  $SU(4)_{\text{ETC}}$  breaking system. The candidates of condensation are :

$$(4, 1, 1, 1, 2)_0 \times (6, 1, 1, 1, 2)_0 \rightarrow (\bar{4}, 1, 1, 1, 1)_0, \\ : k_4^{(\bar{4}, 1)} = \frac{5}{2}\alpha_{4(\text{ETC})}(\Lambda_2) + \frac{3}{2}\alpha_{2(\text{HC})}(\Lambda_2), \quad (10)$$

$$(6, 1, 1, 1, 2)_0 \times (6, 1, 1, 1, 2)_0 \rightarrow (1, 1, 1, 1, 3)_0, \\ : k_4^{(1, 3)} = 5\alpha_{4(\text{ETC})}(\Lambda_2) - \frac{1}{2}\alpha_{2(\text{HC})}(\Lambda_2), \quad (11)$$

where the representations were labeled by  $(SU(4)_{\text{ETC}}, SU(3)_1, SU(3)_2, SU(2)_L, SU(2)_{\text{HC}})_{U(1)_Y}$  and  $\Delta C_2(4 \times 6 \rightarrow \bar{4}) = 5/2, \Delta C_2(6 \times 6 \rightarrow 1) = 5$  for  $SU(4)_{\text{ETC}}$ .

The channel in Eq.(10) is the MAC rather than Eq.(11) if

$$k_4^{(\bar{4}, 1)} > k_4^{(1, 3)} \Leftrightarrow \alpha_{2(\text{HC})}(\Lambda_2) > \frac{5}{4}\alpha_{4(\text{ETC})}(\Lambda_2), \quad (12)$$

is satisfied at  $\Lambda_2$ . The condensation of Eq.(10) correspond to  $\langle H_2^{\text{ETC}} \rangle = \Lambda_2 \neq 0$  in Sec. 2.1. Once  $k_4^{(\bar{4}, 1)}$  exceeds the critical binding strength  $k_{\text{crit}}$  (as will be discussed in Sec. 3),  $SU(4)_{\text{ETC}}$  breaks down to  $SU(3)_{\text{ETC}}$ , and as a result quark/lepton sector is

divided below  $\Lambda_2$  as

$$\begin{aligned} \mathcal{Q}_L &\rightarrow \begin{cases} (1, 3, 1, 2, 1)_{1/6} : q_{1L} = (u, d)_L \\ (1, 3, 1, 2, 1)_{1/6} : q_{2L} = (c, s)_L, \\ (3, 3, 1, 2, 1)_{1/6} : \mathcal{Q}_L'' \end{cases}, \quad \mathcal{L}_L \rightarrow \begin{cases} (1, 1, 1, 1, 1)_{-1/2} : l_{1L} = (\nu_e, e)_L \\ (1, 1, 1, 1, 1)_{-1/2} : l_{2L} = (\nu_\mu, \mu)_L \\ (3, 1, 1, 1, 1)_{-1/2} : \mathcal{L}_L'' \end{cases} \quad (13) \\ \mathcal{U}_R &\rightarrow \begin{cases} (1, 3, 1, 1, 1)_{2/3} : u_R \\ (1, 3, 1, 1, 1)_{2/3} : c_R \\ (3, 3, 1, 1, 1)_{2/3} : \mathcal{U}_R'' \end{cases}, \end{aligned}$$

$$\mathcal{D}_R \rightarrow \begin{cases} (1, 3, 1, 1, 1)_{-1/3} : d_R \\ (1, 3, 1, 1, 1)_{-1/3} : s_R \\ (3, 3, 1, 1, 1)_{-1/3} : \mathcal{D}_R'' \end{cases}, \quad \mathcal{E}_R \rightarrow \begin{cases} (1, 1, 1, 1, 1)_{-1} : e_R \\ (1, 1, 1, 1, 1)_{-1} : \mu_R \\ (3, 1, 1, 1, 1)_{-1} : \mathcal{E}_R'' \end{cases} \quad (15)$$

and the remaining fields of  $\psi_R, \psi_R'$  are

$$\psi_R : \{(1, 1, 1, 1, 2)_0, (3, 1, 1, 1, 2)_0\}, \quad \psi_R' : \{(1, 1, 1, 1, 1)_0, (\bar{3}, 1, 1, 1, 1)_0\}, \quad (16)$$

where we labeled the representations by  $(SU(3)_{\text{ETC}}, SU(3)_1, SU(3)_2, SU(2)_L, SU(2)_{\text{HC}})_{U(1)_Y}$ .

### 2.2.3 Realization of $SU(3)_{\text{ETC}}$ breaking down to $SU(2)_{\text{TC}}$

Finally,  $(1, 1, 1, 1, 2)_0, (3, 1, 1, 1, 2)_0, (1, 1, 1, 1, 1)_0, (\bar{3}, 1, 1, 1, 1)_0$  and the  $SU(2)_{\text{HC}}$  gauge interaction contributes the  $SU(3)_{\text{ETC}}$  breaking system. The candidates of condensation are :

$$\begin{aligned} (3, 1, 1, 1, 2)_0 \times (3, 1, 1, 1, 2)_0 &\rightarrow (\bar{3}, 1, 1, 1, 1)_0, \\ &: k_3^{(\bar{3}, 1)} = \frac{4}{3}\alpha_{3(\text{ETC})}(\Lambda_3) + \frac{3}{2}\alpha_{2(\text{HC})}(\Lambda_3), \end{aligned} \quad (17)$$

$$\begin{aligned} (3, 1, 1, 1, 2)_0 \times (\bar{3}, 1, 1, 1, 1)_0 &\rightarrow (1, 1, 1, 1, 2)_0, \\ &: k_3^{(1, 2)} = \frac{8}{3}\alpha_{3(\text{ETC})}(\Lambda_3), \end{aligned} \quad (18)$$

where the representations were labeled by  $(SU(3)_{\text{ETC}}, SU(3)_1, SU(3)_2, SU(2)_L, SU(2)_{\text{HC}})_{U(1)_Y}$  and  $\Delta C_2(3 \times 3 \rightarrow \bar{3}) = 4/3, \Delta C_2(3 \times \bar{3} \rightarrow 1) = 8/3$  for  $SU(3)_{\text{ETC}}$ .

The channel in Eq.(17) is the MAC rather than Eq.(18) if

$$k_3^{(\bar{3}, 1)} > k_3^{(1, 2)} \iff \alpha_{2(\text{HC})}(\Lambda_3) > \frac{8}{9}\alpha_{3(\text{ETC})}(\Lambda_3), \quad (19)$$

is satisfied at  $\Lambda_3$ . The condensation of Eq.(17) correspond to  $\langle H_3^{\text{ETC}} \rangle \neq 0$  in Sec. 2.1.

Once  $k_4^{(\bar{3},1)}$  exceeds the critical binding strength  $k_{\text{crit}}$  (as will be discussed in Sec. 3),  $SU(3)_{\text{ETC}}$  breaks down to  $SU(2)_{\text{TC}}$ , and as a result, quark/lepton sector is divided below  $\Lambda_3$  as

$$\mathcal{Q}_L \rightarrow \begin{cases} (1, 3, 1, 2, 1)_{1/6} : q_{1L} = (u, d)_L \\ (1, 3, 1, 2, 1)_{1/6} : q_{2L} = (c, s)_L \\ (1, 3, 1, 2, 1)_{1/6} : q_{3L} = (t, b)_L \\ (2, 3, 1, 2, 1)_{1/6} : Q_L^a = (U^a, D^a)_L \end{cases}, \quad \mathcal{L}_L \rightarrow \begin{cases} (1, 1, 1, 1, 1)_{-1/2} : l_{1L} = (\nu_e, e)_L \\ (1, 1, 1, 1, 1)_{-1/2} : l_{2L} = (\nu_\mu, \mu)_L \\ (1, 1, 1, 1, 1)_{-1/2} : l_{3L} = (\nu_\tau, \tau)_L \\ (2, 1, 1, 1, 1)_{-1/2} : L_L^a = (N^a, E^a)_L \end{cases} \quad (20)$$

$$\mathcal{U}_R \rightarrow \begin{cases} (1, 3, 1, 1, 1)_{2/3} : u_R \\ (1, 3, 1, 1, 1)_{2/3} : c_R \\ (1, 3, 1, 1, 1)_{2/3} : t_R \\ (2, 3, 1, 1, 1)_{2/3} : U_R^a \end{cases}, \quad (21)$$

$$\mathcal{D}_R \rightarrow \begin{cases} (1, 3, 1, 1, 1)_{-1/3} : d_R \\ (1, 3, 1, 1, 1)_{-1/3} : s_R \\ (1, 3, 1, 1, 1)_{-1/3} : b_R \\ (2, 3, 1, 1, 1)_{-1/3} : D_R^a \end{cases}, \quad \mathcal{E}_R \rightarrow \begin{cases} (1, 1, 1, 1, 1)_{-1} : e_R \\ (1, 1, 1, 1, 1)_{-1} : \mu_R \\ (1, 1, 1, 1, 1)_{-1} : \tau_R \\ (2, 1, 1, 1, 1)_{-1} : E_R^a \end{cases} \quad (22)$$

and the remaining fields of  $\psi, \psi'$  are

$$2 \times (1, 1, 1, 1, 2)_0, \quad (23)$$

$$2 \times (1, 1, 1, 1, 1)_0, \quad (24)$$

$$N_R^a \equiv (2, 1, 1, 1, 1)_0 \quad (25)$$

where the representations were labeled by  $(SU(2)_{\text{TC}}, SU(3)_1, SU(3)_2, SU(2)_L, SU(2)_{\text{HC}})_{U(1)_Y}$ . We identify  $(2, 1, 1, 1, 1)_0$ -field with right-handed techni-neutrino, so that technilepton condensation preserves the custodial  $SU(2)$ ,  $\langle \bar{N}_R N_L \rangle = \langle \bar{E}_R E_L \rangle$ .

As we noted before,  $\omega_R$  contributes only to the running behavior of the  $SU(2)_{\text{HC}}$  gauge coupling, and the largest possible number of  $\omega$  is  $N_\omega \leq 10$  in order to keep the asymptotic freedom. As  $N_\omega$  increases, the hierarchy among  $\Lambda_i$ s becomes large. As we discuss in Sec.3, if we take  $N_\omega = 2$  as Ref. [19], all  $\Lambda_i$ s become nearly degenerate. We shall take the largest possible value  $N_\omega = 10$  in order to maximize the hierarchy.

Since the  $SU(2)_{\text{HC}}$  is confined at a scale near  $\Lambda_3$ , after ETC gauge group breaks down to TC gauge group, we have ordinary SM quarks/leptons and one family technifermions except for the  $SU(3)_1 \times SU(3)_2$  instead of  $SU(3)_{\text{QCD}}$ .

### 3 Criticality of the successive ETC breakings

Now we come to the discussions on the criticality of the MAC identified in the previous section. By taking account of the criticality condition and the running effect of the ETC gauge couplings at each breaking stage, we obtain definite value of  $\Lambda_2$  and  $\Lambda_3$  once  $\Lambda_1 (= 1000 \text{ TeV})$  is fixed as an input, the lowest allowed scale from the  $K_0 \bar{K}_0$ -mixing. This is in contrast to Ref [18, 19] which did not impose the criticality condition for the breaking of the  $SU(5)_{\text{ETC}} \rightarrow SU(4)_{\text{ETC}}$  and the running effect of the ETC gauge couplings at each breaking stage, and hence treated  $\Lambda_2$  and  $\Lambda_3$  as adjustable parameters.

Several analyses based on the ladder SD equation show that the critical binding strength for the MAC condensation for breaking ETC gauge symmetries is  $k_{\text{crit}} = 2\pi/3$  [35], so that each  $k_N^{(A,B)}$  for the MAC should be larger than  $k^{\text{crit}}$ :

$$k_N^{(A,B)} > k^{\text{crit}} = \frac{2\pi}{3}. \quad (26)$$

Our assumption about the running of ETC gauge coupling is  $k_5^{(5,1)} \simeq k_{\text{crit}}$  at  $\Lambda_1 = 1000 \text{ TeV}$  which corresponds to

$$\alpha_{5(\text{ETC})}(\Lambda_1) = 0.436. \quad (27)$$

There is of course possible error  $1 - 20\%$  [36] of the estimation of  $k^{\text{crit}}$  due to the ladder approximation. Even if we take account of possible  $30\%$  ambiguity of the critical coupling,  $\alpha_{5(\text{ETC})}(\Lambda_1)$  could be lowered only to  $\alpha_{5(\text{ETC})}(\Lambda_1) = 0.31$ . This is compared with the value  $\alpha_{5(\text{ETC})}(\Lambda_1) = 0.1$  used in Ref. [18, 19].

Now we discuss the running effect of the ETC coupling on the criticality conditions, starting with  $\alpha_{5(\text{ETC})}(\Lambda_1 = 1000 \text{ TeV}) = 0.436$  as in Eq.(27). The renormalization equation of each ETC gauge coupling are

$$\mu \frac{\partial}{\partial \mu} \alpha_{N(\text{ETC})} = -b_N \alpha_{N(\text{ETC})}^2, \quad (28)$$

$$b_N = \frac{1}{6\pi} \left[ 11N - 2 \left( \frac{1}{2} N_L^f + \frac{1}{2} N_R^f + \frac{N-2}{2} N_R^{\text{asym}} \right) \right] (> 0), \quad (29)$$

where  $N_{L(R)}^f$  is the number of left(right)-handed fermion with fundamental representation under  $SU(N)_{\text{ETC}}$  and  $N_R^{\text{asym}}$  is the number of right handed fermion with antisymmetric second rank representation under  $SU(N)_{\text{ETC}}$ .  $(N_L^f, N_R^f, N_R^{\text{asym}})$  for each ETC gauge group is

$$(N_L^f, N_R^f, N_R^{\text{asym}}) = \begin{cases} (8, 7, 3) & \text{for } N = 5 \\ (8, 10, 2) & \text{for } N = 4 \\ (8, 9, 1) & \text{for } N = 3 \\ (8, 8, 0) & \text{for } N = 2 \end{cases}. \quad (30)$$

$(N_L^f, N_R^f, N_R^{\text{asym}})$  for  $SU(2)_{\text{HC}}$  gauge group with  $N_\omega = 2$  or 10 is

$$N_\omega = 2 \quad N_\omega = 10$$

$$(N_L^f, N_R^f, N_R^{\text{asym}}) = \begin{cases} (0, 12, 0) & (0, 20, 0) & \text{for } \Lambda_2 < \mu \\ (0, 6, 0) & (0, 14, 0) & \text{for } \Lambda_3 < \mu < \Lambda_2 \\ (0, 4, 0) & (0, 12, 0) & \text{for } \mu < \Lambda_3 \end{cases} . \quad (31)$$

In order to realize the desirable breaking  $SU(4)_{\text{ETC}} \rightarrow SU(3)_{\text{ETC}}$  through the channel in Eq.(10) with Eq.(12) at the scale  $\Lambda_2$  lower than  $\Lambda_1$ , we should take  $SU(2)_{\text{HC}}$  gauge coupling at  $\Lambda_1$  as  $\alpha_{2(\text{HC})}(\Lambda_1) = 0.59$  for  $N_\omega = 2$  case and  $\alpha_{2(\text{HC})}(\Lambda_1) = 0.57$  for  $N_\omega = 10$  case.

Here we comment on  $\Lambda_{2,3}$  in the case of  $N_\omega = 2$  in Ref. [19]. In this case,  $\Lambda_{2,3}$  is given as  $\Lambda_2 = 850 \text{ TeV}$  and  $\Lambda_3 = 500 \text{ TeV}$ , so we obtain no large hierarchy between  $\Lambda_3$  and  $\Lambda_{1,2}$  and hence no large mass difference between the third generation and the second/first generation.

In this paper, instead of  $N_\omega = 2$ , we take  $N_\omega = 10$ , which is the largest possible number of  $\omega$  fulfilling  $N_\omega \leq 10$  in order to keep the asymptotic freedom. Using settings of  $N_\omega = 10$  and  $k_N^{(A,B)}(\Lambda_i) \simeq k_{\text{crit}}$ ,  $\Lambda_{2,3}$  is given by <sup>7</sup>

$$\Lambda_2 = 850 \text{ TeV}, \Lambda_3 = 360 \text{ TeV} . \quad (32)$$

After all the ETC gauge group breakings take place, we are left with  $SU(2)$  TC theory with  $N_f = 8$  as discussed in Sec.2.2. This TC has an intrinsic scale  $\Lambda_{\text{TC}}$  at two-loop level, which is taken as an effective cutoff in the walking/conformal TC [10], and we identify  $\Lambda_{\text{TC}}$  with  $\Lambda_3$ .

$$\Lambda_{\text{TC}} = \Lambda_3 = 360 \text{ TeV} : \quad (33)$$

In the case of  $N_\omega = 10$ , we have  $\alpha_{N(\text{ETC})}, \alpha_{2(\text{HC})}$  at each scale of ETC breakings ( $\Lambda_1 = 1000 \text{ TeV}$ ,  $\Lambda_2 = 850 \text{ TeV}$  and  $\Lambda_3 = 360 \text{ TeV}$ ) as

$$\alpha_{5(\text{ETC})}(\Lambda_1) = \alpha_{4(\text{ETC})}(\Lambda_1) = 0.436 , \quad (34)$$

$$\alpha_{4(\text{ETC})}(\Lambda_2) = \alpha_{3(\text{ETC})}(\Lambda_2) = 0.476 , \quad (35)$$

$$\alpha_{3(\text{ETC})}(\Lambda_3) = \alpha_{(\text{TC})}(\Lambda_3) = 0.705 , \quad (36)$$

$$\alpha_{2(\text{HC})}(\Lambda_1) = 0.59 , \quad (37)$$

$$\alpha_{2(\text{HC})}(\Lambda_2) = 0.596 , \quad (38)$$

$$\alpha_{2(\text{HC})}(\Lambda_3) = 0.761 . \quad (39)$$

The running behavior of both the ETC/TC gauge couplings and the binding strengths for each stage of ETC breakings are shown in Fig. 1 for the case of  $N_\omega = 10$ .

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<sup>7</sup> If we take account of possible 30% ambiguity of  $k_{\text{crit}}$  and  $\Lambda_1 = 1000 \text{ TeV}$ , we obtain  $\Lambda_2 = 360 \text{ TeV}$  and  $\Lambda_3 = 150 \text{ TeV}$  for  $N_\omega = 10$  case. (This possible 30% ambiguity of  $k_{\text{crit}}$  will produce  $\Lambda_2 = 400 \text{ TeV}$  and  $\Lambda_3 = 220 \text{ TeV}$  for  $\Lambda_1 = 1000 \text{ TeV}$  in the case of  $N_\omega = 2$ .)

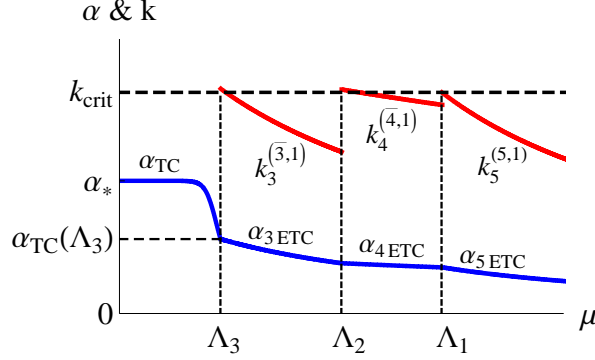


Figure 1: Running behavior of both the ETC/TC gauge couplings and the binding strengths for each stage of ETC breakings for the case of  $N_\omega = 10$  ( $\Lambda_1 = 1000$  TeV,  $\Lambda_2 = 850$  TeV, and  $\Lambda_3 = \Lambda_{\text{TC}} = 360$  TeV.).  $\alpha_* = 2\pi/5$  is the BZ-IRFP for  $SU(2)$  gauge theory with  $N_f = 8$ . Each binding strength is  $k_5^{(5,1)}$  in Eq.(3),  $k_4^{(4,1)}$  in Eq.(10) and  $k_3^{(3,1)}$  in Eq.(17) with the upper dashed line being the critical value  $k_{\text{crit}} = 2\pi/3$ .  $\alpha_{\text{TC}}(\Lambda_{\text{TC}}) = \alpha_{\text{TC}}(\Lambda_3) = 0.705$ .

Now that all ETC breaking scales are fixed uniquely, we come to the discussion on the third generation mass due to the TC condensate through ETC-induced four-fermion interactions:

$$\mathcal{L}_{\text{ETC}}^{\text{TC-3rd}}|_{\text{mass}} = -G_3^{\text{ETC}} \left[ \left( \bar{U}_R \gamma^\mu t_R + \bar{D}_R \gamma^\mu b_R \right) \left( \bar{q}_{3L} \gamma_\mu Q_L \right) \right] + [\text{h.c.}], \quad (40)$$

where

$$G_3^{\text{ETC}} = \frac{g_{3(\text{ETC})}^2}{2M_3^2} = \frac{1}{2\Lambda_3^2} = \frac{4\pi\alpha_{3(\text{ETC})}(\Lambda_3)}{2M_3^2}, \quad (41)$$

and  $M_3 = g_{3(\text{ETC})}\Lambda_3$  is the mass of the broken  $SU(3)$  ETC gauge boson. Now the  $SU(2)$  TC with  $N_f = 8$  is a walking/conformal theory where the two-loop  $\beta$ -function possesses the BZ-IRFP  $\alpha_* = 2\pi/5$  which is lower than the critical coupling evaluated in the ladder SD equation:  $\alpha_* < \alpha^{\text{crit}} = \pi/(3C_2(F))$ . However there is some ambiguity in evaluation of the critical coupling up to 1-20% in the ladder approximation [36]. In fact the critical coupling  $\alpha^{\text{crit}}$  decreases by 20% when we define the critical coupling such that an anomalous dimension is  $\gamma_m = 1$  at two-loop level [10]. So we may expect that the present TC triggers the technifermion condensate by its own dynamics.

After Fiertz rearrangement of Eq.(40), we have the third generation quark mass given by

$$m_{3\text{rd}} = \frac{4\pi\alpha_{3(\text{ETC})}(\Lambda_3)}{2M_3^2} \langle \bar{Q}Q \rangle_{M_3}, \quad (42)$$

where  $\langle \bar{Q}Q \rangle_{M_3} = \langle \bar{U}_R Q_L \rangle_{M_3} = \langle \bar{D}_R Q_L \rangle_{M_3}$  is the technifermion condensate given by

$$\begin{aligned} \langle \bar{Q}Q \rangle_{M_3} &= -\frac{N_{\text{TC}}}{4\pi^2} \int_0^{M_3^2} dx \frac{x\Sigma(x)}{x + \Sigma(x)^2} \\ &= -\frac{N_{\text{TC}}}{4\pi^2} \left[ \frac{2}{\gamma_m^{(\text{TC})}} \left( \frac{M_3}{m_{\text{TC}}} \right)^{\gamma_m^{(\text{TC})}} + 1 - \ln 2 \right] m_{\text{TC}}^3, \end{aligned} \quad (43)$$

for the dynamical mass function  $\Sigma(x)$  of the technifermion parameterized as

$$\Sigma(x) \sim \begin{cases} m_{\text{TC}} \left( \frac{x}{m_{\text{TC}}^2} \right)^{\frac{1}{2}\gamma_m^{(\text{TC})}-1} & \text{for } x > m_{\text{TC}}^2 \\ m_{\text{TC}} & \text{for } x < m_{\text{TC}}^2 \end{cases}. \quad (44)$$

with the anomalous dimension  $\gamma_m^{(\text{TC})} \simeq 1$  for the walking/conformal  $SU(2)_{\text{TC}}$  technicolor. Thus we have  $m_{3\text{rd}}$

$$\begin{aligned} m_{3\text{rd}} &\simeq \frac{4\pi\alpha_{3(\text{ETC})}(\Lambda_3)}{2M_3^2} \times \frac{m_{\text{TC}}^2 M_3}{\pi^2} \\ &\simeq 0.1 \text{ (GeV)} \times \left( \frac{m_{\text{TC}}}{500 \text{ (GeV)}} \right)^2 \times \left( \frac{360 \text{ (TeV)}}{\Lambda_3} \right), \end{aligned} \quad (45)$$

where we have used Eq.(36) and  $M_3^2 = 4\pi\alpha_{3(\text{ETC})}(\Lambda_3) \times \Lambda_3^2$ , with a typical value of the technifermion mass in the one-family TC model ( $N_{\text{TC}} = 2, N_f = 8$ ) being  $m_{\text{TC}} \simeq 500\text{GeV}$  which corresponds to  $F_\pi^2 \simeq (246\text{GeV})^2/(N_f/2)$  (see, e.g., Eq.(118)-(121)). Thus we can only have small mass for the third generation if the TC is the only origin of the EWSB in this type of ETC.

## 4 The criticality of the EWSB

Let us now discuss the EWSB in the top-mode ETC where both quarks and techniquarks having the same topcolor  $SU(3)_1$  are put in the same representation of ETC.

As to the topcolor breaking, we assume that  $\langle \Phi \rangle = \Lambda_C \neq 0$ , ( $\Lambda_3 > \Lambda_C$ ) trigger the topcolor breaking as

$$\begin{aligned} SU(3)_1 &\times SU(3)_2, \\ &\downarrow \Lambda_C \\ &SU(3)_{\text{QCD}}, \end{aligned} \quad (46)$$

where  $SU(3)_1$  is stronger than  $SU(3)_2$  and the gauge coupling of  $SU(3)_1 \times SU(3)_2$  are given by  $h_1$  and  $h_2$ , respectively. This topcolor breaking generates the 8 massive gauge



bosons ( colorons ) and 8 massless gauge bosons ( gluons ). The colorons mass  $M_C$  is given by

$$M_C = \sqrt{h_1^2 + h_2^2} \Lambda_C, \quad (47)$$

and the  $SU(3)_{\text{QCD}}$  gauge coupling is given by

$$g_{\text{QCD}} = h_1 \sin \theta = h_2 \cos \theta, \quad (48)$$

where we defined the mixing angle  $\theta$  as

$$\cot \theta \equiv \frac{h_1}{h_2} (> 1). \quad (49)$$

After all the ETC and topcolor breakings occurred ( A mechanism of ETC breaking is shown in Sec. 3), we obtain the  $SU(2)_{\text{TC}} \times SU(3)_{\text{QCD}} \times SU(2)_L \times U(1)_Y$  invariant four fermion interaction :

$$\mathcal{L}^{4f} = \sum_{i,j} \mathcal{L}_{\text{ETC}}^{i-j} + \sum_{i,j} \mathcal{L}_{\text{topC}}^{i-j}, \quad (50)$$

where  $i, j$  is TC or SM and for example  $\mathcal{L}_{\text{ETC}}^{\text{TC-SM}}$  represents four fermion interactions between technifermions and SM-fermions via massive ETC gauge bosons. For the moment we shall concentrate on the four-fermion interactions which lead to the diagonal mass of the quarks/leptons in Eq.(50):

$$\begin{aligned} \mathcal{L}_{\text{ETC}}^{\text{TC-SM}}|_{\text{mass}} = & -G_1^{\text{ETC}} \left[ \left( \bar{U}_R \gamma^\mu u_R + \bar{D}_R \gamma^\mu d_R \right) \left( \bar{q}_{1L} \gamma_\mu Q_L \right) + \left( \bar{t}_R \gamma^\mu u_R \right) \left( \bar{q}_{1L} \gamma_\mu q_{3L} \right) \right] \\ & -G_2^{\text{ETC}} \left[ \left( \bar{U}_R \gamma^\mu c_R + \bar{D}_R \gamma^\mu s_R \right) \left( \bar{q}_{2L} \gamma_\mu Q_L \right) + \left( \bar{t}_R \gamma^\mu c_R \right) \left( \bar{q}_{2L} \gamma_\mu q_{3L} \right) \right] \\ & -G_3^{\text{ETC}} \left[ \left( \bar{U}_R \gamma^\mu t_R + \bar{D}_R \gamma^\mu b_R \right) \left( \bar{q}_{3L} \gamma_\mu Q_L \right) + \frac{1}{3} \left( \bar{t}_R \gamma^\mu t_R \right) \left( \bar{q}_{3L} \gamma_\mu q_{3L} \right) \right] \\ & - \sum_i G_i^{\text{ETC}} \left( \bar{E}_R \gamma^\mu e_{iR} \right) \left( \bar{l}_{iL} \gamma_\mu L_L \right) + [\text{h.c.}], \end{aligned} \quad (51)$$

where

$$G_i^{\text{ETC}} = \frac{c_i \times 4\pi \alpha_{N(\text{ETC})}}{2M_i^2}, \quad \left( i = 1, 2, 3; c_1 = 2, c_2 = \frac{3}{2}, c_3 = 1 \right), \quad (52)$$

$g_{N(\text{ETC})}$  is the  $SU(N)_{\text{ETC}}$  gauge coupling at  $\Lambda_i$ . From Sec. 3, our hierarchy reads  $\Lambda_1 \gg \Lambda_2 \gg \Lambda_3$ . The sideway ETC gauge bosons mass is given by

$$M_i = g_{N(\text{ETC})} \Lambda_i. \quad (53)$$

Let us discuss the criticality of the top quark condensate. We concentrate on  $(\bar{f}f)^2$ -type four fermion interaction part in Eq.(50):

$$\mathcal{L}^{4f}|_{\text{self}} = \sum_{i=1,2,3} \left[ \frac{1}{6-i} G_i^{\text{ETC}} \left( (\bar{q}_{iL} u_{iR})^2 + (\bar{q}_{iL} d_{iR})^2 + (\bar{l}_{iL} e_{iR})^2 \right) + G_{\text{topC}} \left( (\bar{q}_{iL} u_{iR})^2 + (\bar{q}_{iL} d_{iR})^2 \right) \right], \quad (54)$$

where

$$G_{\text{topC}} = \frac{h_1^2 \cos^2 \theta}{4M_C^2} = \frac{4\pi\alpha_{\text{QCD}}}{4M_C^2} \cot^2 \theta = \frac{4\pi}{4M_C^2} \cdot \kappa_3, \quad (55)$$

and

$$\kappa_3 \equiv \alpha_{\text{QCD}} \cot^2 \theta = \alpha_{SU(3)_1} \cos^2 \theta = \alpha_{SU(3)_2} \cos^2 \theta \cot^2 \theta, \quad (56)$$

and  $\alpha_a \equiv g_a^2/4\pi$  ( $a = \text{QCD}, SU(3)_1, SU(3)_2$ ).

Now, we consider the gap equation for the four fermion interaction Eq.(54):

$$1 = \left[ \frac{1}{6-i} G_i^{\text{ETC}} + G_{\text{topC}} \right] \times \frac{2N_c M_C^2}{4\pi^2} \left[ 1 - \frac{m_{\text{dyn}}^2}{M_C^2} \ln \frac{M_C^2}{m_{\text{dyn}}^2} \right], \quad (\text{for quarks}), \quad (57)$$

$$1 = \left[ \frac{1}{6-i} G_i^{\text{ETC}} \right] \times \frac{2M_C^2}{4\pi^2} \left[ 1 - \frac{m_{\text{dyn}}^2}{M_C^2} \ln \frac{M_C^2}{m_{\text{dyn}}^2} \right], \quad (\text{for leptons}), \quad (58)$$

where  $N_c (= 3)$  is the number of colors, and  $m_{\text{dyn}}$  is the dynamical mass of each fermion. We define the dimensionless four fermion coupling  $g_{fi}$ , ( $f$  stands for the SM fermions) as

$$g_{ui} \equiv \left[ G_i^{\text{ETC}} + G_{\text{topC}} \right] \frac{2N_c M_C^2}{4\pi^2} = N_c \cdot g_i^{\text{ETC}} + N_c \cdot \frac{\kappa_3}{2\pi}, \quad (59)$$

$$g_{di} \equiv \left[ G_i^{\text{ETC}} + G_{\text{topC}} \right] \frac{2N_c M_C^2}{4\pi^2} = N_c \cdot g_i^{\text{ETC}} + N_c \cdot \frac{\kappa_3}{2\pi}, \quad (60)$$

$$g_{ei} \equiv \left[ G_i^{\text{ETC}} \right] \frac{2M_C^2}{4\pi^2} = g_i^{\text{ETC}}, \quad (61)$$

where

$$g_i^{\text{ETC}} = G_i^{\text{ETC}} \cdot \frac{2M_C^2}{4\pi^2} = \frac{c_i \alpha_{N(\text{ETC})}}{\pi} \cdot \left( \frac{M_C}{M_i} \right)^2. \quad (62)$$

We can realize the situation that the top quark is the only SM fermion to condense, only if the following conditions are met:

$$g_t > g_t^{\text{crit}}, \quad g_{b,\tau} < g^{\text{crit}}, \quad (63)$$

$$g_{\tilde{f}} < g^{\text{crit}}, \quad (\tilde{f} = u, d, c, s, e, \mu). \quad (64)$$

As we discussed in Sec. 3 we have the hierarchical ETC breaking scale, so that we have a hierarchy of  $g_i^{\text{ETC}}$

$$\begin{aligned} g_3^{\text{ETC}} &> g_2^{\text{ETC}} = g_3^{\text{ETC}} \times \frac{3}{4} \cdot \frac{c_2}{c_3} \cdot \left( \frac{M_3}{M_2} \right)^2 \\ &> g_1^{\text{ETC}} = g_3^{\text{ETC}} \times \frac{3}{5} \cdot \frac{c_1}{c_3} \cdot \left( \frac{M_3}{M_1} \right)^2, \end{aligned} \quad (65)$$

i.e., the condensate of the third generation quarks/leptons is favored to that of others. Then we concentrate on Eq.(63).

In the NJL case  $g^{\text{crit}}$  is  $g^{\text{crit}} = 1$  in Eq.(63), however, dynamics in the present case is the gauged NJL model ( SM gauge + NJL ). We recall the critical coupling (critical line) in the gauged NJL model [37]:

$$g_f^{\text{crit}} = \frac{1}{4} \left( 1 + \sqrt{1 - \frac{\alpha_f}{\pi/3}} \right)^2, \quad (66)$$

where

$$\alpha_{f=t,c,u}(\mu) = \frac{4}{3} \alpha_{\text{QCD}}(\mu) + \frac{1}{9} \alpha_Y(\mu), \quad (67)$$

$$\alpha_{f=b,s,d}(\mu) = \frac{4}{3} \alpha_{\text{QCD}}(\mu) - \frac{1}{18} \alpha_Y(\mu), \quad (68)$$

$$\alpha_{f=e,\mu,\tau}(\mu) = \frac{1}{2} \alpha_Y(\mu), \quad (69)$$

In order to obtain the top quark condensation,  $\kappa_3$  should satisfy Eq.(63) which read:

$$\kappa_3 + 2\pi \cdot g_3^{\text{ETC}} > \frac{2\pi}{N_c} \cdot g_t^{\text{crit}}, \quad (70)$$

$$\kappa_3 + 2\pi \cdot g_3^{\text{ETC}} < \frac{2\pi}{N_c} \cdot g_b^{\text{crit}}, \quad (71)$$

$$g_3^{\text{ETC}} < g_\tau^{\text{crit}}, \quad (72)$$

In the present case

$$g_3^{\text{ETC}} \simeq 5.0 \times 10^{-6} \times \left( \frac{M_C}{5 \text{ (TeV)}} \right)^2 \times \left( \frac{360 \text{ (TeV)}}{\Lambda_3} \right)^2, \quad (73)$$

so we can neglect  $g_3^{\text{ETC}}$  in Eq.(70), (71) and (72). The coloron mass is constrained by the experiment  $M_C / \cot \theta > 837 \text{ GeV}$  [33], which implies  $M_C \gtrsim 3 \text{ TeV}$  in our case (see Eq.(79)). We shall take

$$M_C = 5 \text{ TeV}. \quad (74)$$

In order to trigger the top quark condensation in the present model,  $\kappa_3$  must satisfy

$$\kappa_3 > \frac{2\pi}{N_c} \cdot g_t^{\text{crit}} \quad \text{and} \quad \kappa_3 < \frac{2\pi}{N_c} \cdot g_b^{\text{crit}}, \quad (75)$$

where in the case of  $M_C \simeq 5 \text{ TeV}$ ,

$$g_t^{\text{crit}}(M_C) \simeq 0.942 \quad , \quad g_b^{\text{crit}}(M_C) \simeq 0.943, \quad (76)$$

which correspond to

$$\alpha_t(M_C) \simeq 0.119 \quad , \quad \alpha_b(M_C) \simeq 0.117, \quad (77)$$

where we have used inputs:  $\alpha_Y(M_Z) = 0.0101684 \pm 0.0000014$ ,  $\alpha_{\text{QCD}}(M_Z) = 0.1176 \pm 0.0020$  [38]. Eq.(75) and (76) show a constraint on  $\kappa_3$  as

$$1.973 < \kappa_3 < 1.975. \quad (78)$$

From Eq.(56), this result shows

$$22.22 < \cot^2 \theta < 22.25, \quad \text{and} \quad 2.062 < \alpha_{SU(3)_1}(M_C) < 2.064. \quad (79)$$

Now we discuss the criticality of the technifermion condensate. In this model, technifermions have topcolor charge as shown in Table. 1, so that the technifermion condensate is triggered at the scale  $\mu$  if  $\alpha_{U/D/E/N}(\mu) > \pi/3$  is satisfied, where  $\alpha_{U/D/E/N}(\mu)$  is given by

$$\alpha_U(\mu) = \frac{3}{4} \alpha_{\text{TC}}(\mu) + \frac{4}{3} \alpha_{SU(3)_1}(\mu) + \frac{1}{9} \alpha_Y(\mu) \quad (80)$$

$$\alpha_D(\mu) = \frac{3}{4} \alpha_{\text{TC}}(\mu) + \frac{4}{3} \alpha_{SU(3)_1}(\mu) - \frac{1}{18} \alpha_Y(\mu), \quad (81)$$

$$\alpha_E(\mu) = \frac{3}{4} \alpha_{\text{TC}}(\mu) + \frac{1}{2} \alpha_Y(\mu), \quad (82)$$

$$\alpha_N(\mu) = \frac{3}{4} \alpha_{\text{TC}}(\mu). \quad (83)$$

From Eq.(79) and Eq.(36) we estimate  $\alpha_{SU(3)_1}(\mu > M_C)$  and  $\alpha_{\text{TC}}(\mu < \Lambda_3)$ , respectively. The  $U(1)_Y$  contribution is negligible. Then we find that the combined coupling of TC and  $SU(3)_1$  is rather strong already at  $\mu = \Lambda_3$ :  $\alpha_U(\Lambda_3) \simeq 0.972 \sim \pi/3$ . In fact we find  $\alpha_{U/D}(\mu) > \pi/3$  at  $\mu \simeq 80 \text{ TeV}$  which implies that the dynamical mass of the techniquark and hence the decay constant  $f_{\pi_T}$  is on this order  $f_{\pi_T} \sim m_{\text{TC}} \simeq 80 \text{ TeV}$ , which is extremely large compared with the weak scale and is a disaster.

Therefore, the framework in this section cannot give us a desirable result. In order to avoid this problem, we should change the topcolor charge assignment of the technifermions, so that the technicolor criticality could be unaffected by the strong topcolor which is required to be near criticality for triggering the top quark condensate. In the Section 5, we consider such a new TC2 model, although an explicit ETC model building is not attempted in this paper.

field	$SU(2)_{\text{TC}}$	$SU(3)_1$	$SU(3)_2$	$SU(2)_L$	$U(1)_{Y1}$	$U(1)_{Y2}$
$Q_L$	2	1	3	2	0	1/6
$U_R$	2	1	3	1	0	2/3
$D_R$	2	1	3	1	0	-1/3
$L_L$	2	1	1	1	0	-1/2
$E_R$	2	1	1	1	0	-1
$N_R$	2	1	1	1	0	0
$q_{iL}$	1	3	1	2	1/6	0
$u_{iR}$	1	3	1	1	2/3	0
$d_{iR}$	1	3	1	1	-1/3	0
$l_{iL}$	1	1	1	2	-1/2	0
$e_{iR}$	1	1	1	1	-1	0

Table 2: Particle contents in the twisted flavor-universal TC2 model.

## 5 Twisted flavor-universal TC2

As discussed in Sec. 4, we should change the topcolor charge assignment of the techniquarks. In this section, as the first step to build an explicit ETC model having such a topcolor assignment, we here consider a new type TC2 model, twisted flavor-universal TC2 model, in order to forbid a techniquark condensate at too large scale. In addition to the topcolor  $SU(3)_1 \times SU(3)_2$  we here introduce an extended hypercharge sector  $U(1)_{Y1} \times U(1)_{Y2}$  [34] to be spontaneously broken into the SM hypercharge symmetry  $U(1)_Y$ , in such a way that SM fermions carry the flavor-universal  $SU(3)_1 \times U(1)_{Y1}$ , while technifermions do the opposite charges  $SU(3)_2 \times U(1)_{Y2}$ . The charge assignments in this twisted flavor-universal TC2 model are shown in Table. 2.

Since the techniquarks and quarks have different topcolor charges, it is highly non-trivial to put them into a single representation of the ETC. We would need larger picture to unify them. For the moment we shall not try such an explicit model building but discuss possible consequences if the ETC type interactions communicate the SM fermions and the technifermions. Such a possibility may also be realized by the composite model for the SM fermions and the technifermions [3]. All the setting of symmetry breakings is made analogously to the ETC breakings of the model studied in the Sec. 2 and Sec. 3. Such an approach is the same as the conventional TC2 model building.

The symmetry breaking is assumed as follows :

$$\begin{aligned}
& \text{ETC}_1 \times SU(3)_1 \times SU(3)_2 \times SU(2)_L \times U(1)_{Y1} \times U(1)_{Y2} \\
& \quad \downarrow \Lambda_1 \\
& \text{ETC}_2 \times SU(3)_1 \times SU(3)_2 \times SU(2)_L \times U(1)_{Y1} \times U(1)_{Y2} \\
& \quad \downarrow \Lambda_2 \\
& \text{ETC}_3 \times SU(3)_1 \times SU(3)_2 \times SU(2)_L \times U(1)_{Y1} \times U(1)_{Y2} \\
& \quad \downarrow \Lambda_3 \\
& SU(2)_{\text{TC}} \times SU(3)_1 \times SU(3)_2 \times SU(2)_L \times U(1)_{Y1} \times U(1)_{Y2}, \\
& \quad \downarrow \Lambda_C < \Lambda_3 \\
& SU(2)_{\text{TC}} \times \quad SU(3)_{\text{QCD}} \quad \times SU(2)_L \times \quad U(1)_Y,
\end{aligned} \tag{84}$$

where we do not specify each ETC gauge group and dynamical mechanism of the ETC breakings in this section. Also, we do not discuss the dynamical mechanism of the topcolor/extended hypercharge breakings. Each gauge coupling of topcolor/extended hypercharge is follows:  $SU(3)_1(SU(3)_2)$  gauge coupling is  $h_1$  ( $h_2$ ) and  $U(1)_{Y1}(U(1)_{Y2})$  gauge coupling is  $g'_{Y1}$  ( $g'_{Y2}$ ), where  $SU(3)_1/U(1)_{Y1}$  is stronger than  $SU(3)_2/U(1)_{Y2}$ . The colorons and  $Z'$  mass  $M_C, M_{Z'}$  are

$$M_C = \sqrt{h_1^2 + h_2^2} \Lambda_C, \tag{85}$$

$$M_{Z'} = \sqrt{g'^2_{Y1} + g'^2_{Y2}} \Lambda_C \tag{86}$$

and the  $SU(3)_{\text{QCD}}$  and  $U(1)_Y$  gauge couplings are given by

$$g_{\text{QCD}} = h_1 \sin \theta = h_2 \cos \theta, \tag{87}$$

$$g'_Y = g'_{Y1} \sin \eta = g'_{Y2} \cos \eta \tag{88}$$

where we defined the mixing angles  $\theta$  and  $\eta$  as

$$\cot \theta \equiv \frac{h_1}{h_2} (> 1) \quad , \quad \cot \eta \equiv \frac{g'_{Y1}}{g'_{Y2}} (> 1). \tag{89}$$

As to the  $\text{ETC}_3 \rightarrow SU(2)_{\text{TC}}$  breaking, the sideways gauge boson mass  $M_3$  is a free parameter at this moment and we assume here  $M_3 \gtrsim M_C = M_{Z'}$  for simplicity.

At  $\Lambda_C$  the topcolor/extended hypercharge gauge groups as well as the ETC group spontaneously break down, so that we have  $(\bar{f}f)^2$ -type four fermion interactions:

$$\begin{aligned}
\mathcal{L}^{4f} = & \sum_{i=1,2,3} \left[ G_i^{\text{ETC}} \left( (\bar{q}_{iL} u_{iR})^2 + (\bar{q}_{iL} d_{iR})^2 + (\bar{l}_{iL} e_{iR})^2 \right) + G_{\text{topC}}^s \left( (\bar{q}_{iL} u_{iR})^2 + (\bar{q}_{iL} d_{iR})^2 \right) \right. \\
& \left. + G_{Z'}^{su} (\bar{q}_{iL} u_{iR})^2 + G_{Z'}^{sd} (\bar{q}_{iL} d_{iR})^2 + G_{Z'}^{sl} (\bar{l}_{iL} e_{iR})^2 \right],
\end{aligned} \tag{90}$$

where  $q_{iL} = (u_i, d_i)_L^T$ ,  $l_{iL} = (\nu_i, e_i)_{iL}$ ,  $u_3 = t$ ,  $d_3 = b$ ,  $\nu_3 = \nu_\tau$ ,  $e_3 = \tau, \dots$ .  $G_i^{\text{ETC}} = 4\pi\alpha_{\text{ETC}_i}(\Lambda_i)/(2M_i^2) = 1/(2\Lambda_i^2)$  ( $i = 1, 2, 3$ ) are the four-fermion couplings of the SM fermions generated by the ETC breaking at each scale  $\Lambda_i$ ,  $G_{\text{topC}}^s$  is the four-fermion coupling generated by the topcolor  $SU(3)_1 \times SU(3)_2$  breaking at  $\Lambda_C$  and  $G_{Z'}^{sf}$  is the four-fermion coupling of the fermions:  $f$ , ( $f = t, b, c, s, u, d, \dots$ ) generated by the extended hypercharge gauge symmetry  $U(1)_{Y1} \times U(1)_{Y2}$  breaking at  $\Lambda_C$ .

We rewrite  $G_i^{\text{ETC}}$ ,  $G_{\text{topC}}^s$  and  $G_{Z'}^s$  by  $g_i^{\text{ETC}}$ ,  $\kappa_3$  and  $\kappa_1$ , respectively

$$G_i^{\text{ETC}} = \frac{2\pi^2}{M_C^2} \cdot g_i^{\text{ETC}} \quad , \quad G_{\text{topC}}^s = \frac{4\pi}{4M_C^2} \cdot \kappa_3 \quad , \quad G_{Z'}^{sf} = \frac{4\pi}{2M_{Z'}^2} \cdot \kappa_1 Y_L^f Y_R^f, \quad (91)$$

where

$$\kappa_3 \equiv \alpha_{\text{QCD}} \cot^2 \theta = \alpha_{SU(3)_1} \cos^2 \theta = \alpha_{SU(3)_2} \cos^2 \theta \cot^2 \theta, \quad (92)$$

$$\kappa_1 \equiv \alpha_Y \cot^2 \eta = \alpha_{Y1} \cos^2 \eta = \alpha_{Y2} \cos^2 \eta \cot^2 \eta, \quad (93)$$

and  $\alpha_m \equiv g_m^2/4\pi$  ( $m = \text{QCD}, SU(3)_1, SU(3)_2$ ) and  $\alpha_n \equiv g_n^2/4\pi$  ( $n = Y, Y1, Y2$ ). We define the dimensionless four-fermion coupling  $g_{fi}$  as

$$g_{u,c,t} \equiv [N_c G_i^{\text{ETC}} + N_c G_{\text{topC}}^s + G_{Z'}^{su}] \frac{2M_C^2}{4\pi^2} = N_c \cdot g_i^{\text{ETC}} + N_c \cdot \frac{\kappa_3}{2\pi} + \frac{1}{9} \cdot \frac{\kappa_1}{\pi}, \quad (94)$$

$$g_{d,s,b} \equiv [N_c G_i^{\text{ETC}} + N_c G_{\text{topC}}^s + G_{Z'}^{sd}] \frac{2M_C^2}{4\pi^2} = N_c \cdot g_i^{\text{ETC}} + N_c \cdot \frac{\kappa_3}{2\pi} - \frac{1}{18} \cdot \frac{\kappa_1}{\pi}, \quad (95)$$

$$g_{e,\mu,\tau} \equiv [G_i^{\text{ETC}} + G_{Z'}^{sl}] \frac{2M_C^2}{4\pi^2} = g_i^{\text{ETC}} + \frac{1}{2} \cdot \frac{\kappa_1}{\pi}, \quad (96)$$

In terms of (94)–(96) we have the conditions that top quark is the only SM fermion to condense,

$$g_t > g_t^{\text{crit}} \quad , \quad g_{b,\tau} < g_{b,\tau}^{\text{crit}}, \quad (97)$$

$$g_{u,d,c,s,e,\mu} < g_{u,d,c,s,e,\mu}^{\text{crit}}, \quad (98)$$

where the critical lines for SM fermions,  $g_{u,c,t}^{\text{crit}}$ , are given in Eq.(66)–(69), which yield

$$\begin{aligned} g_{u,c,t}^{\text{crit}}(M_C) &\simeq 0.942, \\ g_{d,s,b}^{\text{crit}}(M_C) &\simeq 0.943, \\ g_{e,\mu,\tau}^{\text{crit}}(M_C) &\simeq 0.997, \end{aligned} \quad (99)$$

for the values of  $\alpha_{u,c,t} \simeq 0.120$ ,  $\alpha_{d,s,b} \simeq 0.118$ ,  $\alpha_{e,\mu,\tau} \simeq 0.005$  (see Eq.(77)).

The breaking in Eq.(84) generates the hierarchical four-fermion couplings,

$$G_1^{\text{ETC}} < G_2^{\text{ETC}} < G_3^{\text{ETC}}, \quad (100)$$

which implies that the condensation of the third generation fermions are favored to other generations. If we can realize large hierarchical ETC breaking  $\Lambda_{1,2}/\Lambda_3 \gg 1$ , the

condition Eq.(98) can easily fulfilled. Then we concentrate on the condition Eq.(97) in order to consider the criticality of the top quark condensation.

Eq.(97) are explicitly written as

$$2\pi \cdot g_3^{\text{ETC}} + \kappa_3 + \frac{1}{N_c} \cdot \frac{2}{9} \cdot \kappa_1 > \frac{2\pi}{N_c} \cdot g_t^{\text{crit}}, \quad (101)$$

$$2\pi \cdot g_3^{\text{ETC}} + \kappa_3 - \frac{1}{N_c} \cdot \frac{1}{9} \cdot \kappa_1 < \frac{2\pi}{N_c} \cdot g_b^{\text{crit}}, \quad (102)$$

$$g_3^{\text{ETC}} + \frac{1}{2} \cdot \kappa_1 < \pi \cdot g_\tau^{\text{crit}}, \quad (103)$$

where  $g_{t,b,\tau}^{\text{crit}}$  are given by Eq.(99) and the value of  $g_3^{\text{ETC}} = G_3^{\text{ETC}} \cdot 2M_C^2/4\pi = 1/(4\pi^2) \cdot (M_C/\Lambda_3)^2$  is estimated as

$$g_3^{\text{ETC}} \simeq 3.5 \times 10^{-6} \times \left( \frac{M_C}{5 \text{ (TeV)}} \right)^2 \times \left( \frac{360 \text{ (TeV)}}{\Lambda_3} \right)^2, \quad (104)$$

which is negligibly small for  $\Lambda_3 > 360 \text{ TeV}$ . Then the parameter space area ( $\kappa_3, \kappa_1$ ) constrained by Eq.(101), (102) and (103) is represented by the triangle (“gap triangle”) in Fig. 2 in the case of  $\Lambda_C = 1 \text{ TeV}$  for  $M_C = 5 \text{ TeV}$  (See Eq.(74)). The dashed line in Fig. 2 stands for the top quark mass coming from top condensate  $\hat{m}_t$  ( $\simeq m_t^{\text{exp}} = 172 \pm 2.5 \text{ GeV}$ ) determined by the SD gap equation: [39]

$$g_{\hat{m}_t}^{\text{crit}} = \frac{1}{4} \frac{(1 + \sqrt{1 - \alpha_t/(\pi/3)})^2 - (1 - \sqrt{1 - \alpha_t/(\pi/3)})^2 (P + Q)}{1 - P + \frac{3 - \sqrt{1 - \alpha_t/(\pi/3)}}{1 + \sqrt{1 - \alpha_t/(\pi/3)}} Q}, \quad (105)$$

where  $P, Q$  are given by

$$P \equiv \frac{\Gamma(1 - \sqrt{1 - \alpha_t/(\pi/3)}) \Gamma(3/2 + \frac{1}{2} \sqrt{1 - \alpha_t/(\pi/3)})^2 \left( \frac{\hat{m}_t^2}{M_C^2} \right)^{\sqrt{1 - \alpha_t/(\pi/3)}}}{\Gamma(1 + \sqrt{1 - \alpha_t/(\pi/3)}) \Gamma(3/2 - \frac{1}{2} \sqrt{1 - \alpha_t/(\pi/3)})^2},$$

$$Q \equiv \frac{(1 + \sqrt{1 - \alpha_t/(\pi/3)})^2 \hat{m}_t^2}{4(1 - \sqrt{1 - \alpha_t/(\pi/3)}) M_C^2}. \quad (106)$$

Let us consider the criticality of the technifermion condensate in the present model. First, we consider the criticality in the  $\Lambda_C < \mu < \Lambda_3 = \Lambda_{\text{TC}}$ , where the ETC breaking scale  $\Lambda_3$  is identified with the intrinsic scale of the  $\Lambda_{\text{TC}}$  defined by the two-loop beta function [10]. In this region ETC breaks down to TC while topcolor/extended hypercharge gauge symmetry does not. Since  $g_3^{\text{ETC}}$  is small as seen Eq.(104), relevant is the



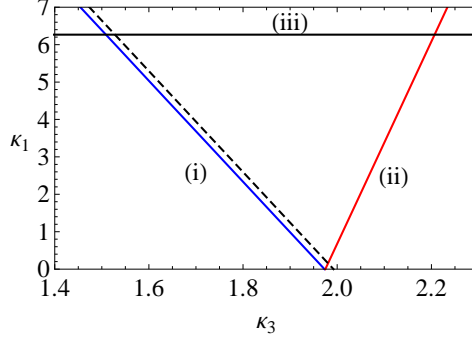


Figure 2: The gap triangle in twisted flavor-universal TC2 model ( $M_C = M_{Z'} = 5$  TeV). The region above (i) represents  $\langle \bar{t}t \rangle \neq 0$ , the region below (ii) represents  $\langle \bar{b}b \rangle \neq 0$  and the region above (iii) represents  $\langle \bar{\tau}\tau \rangle \neq 0$ . Only the top quark forms a condensation in the area (“gap triangle”) enclosed by the lines (i), (ii) and (iii). The dashed line stands for the solution line of the gauged NJL model [39] for a finite dynamical mass of the top quark  $m_t \simeq 170$  GeV and  $M_C = 5$  TeV in Eq.(105).

gauge dynamics described by  $\alpha_{U,D,E,N}(\mu)$ :

$$\alpha_U(\mu) = \frac{3}{4}\alpha_{\text{TC}}(\mu) + \frac{4}{3}\alpha_{SU(3)_2}(\mu) + \frac{1}{9}\alpha_{Y2}(\mu) \quad (107)$$

$$\alpha_D(\mu) = \frac{3}{4}\alpha_{\text{TC}}(\mu) + \frac{4}{3}\alpha_{SU(3)_2}(\mu) - \frac{1}{18}\alpha_{Y2}(\mu), \quad (108)$$

$$\alpha_E(\mu) = \frac{3}{4}\alpha_{\text{TC}}(\mu) + \frac{1}{2}\alpha_{Y2}(\mu), \quad (109)$$

$$\alpha_N(\mu) = \frac{3}{4}\alpha_{\text{TC}}(\mu). \quad (110)$$

Now we observe that  $\alpha_{\text{TC}}(\mu) < \alpha_{\text{TC}}(\Lambda_C) = 1.18$  which is estimated by two-loop running of the  $\alpha_{\text{TC}}(\mu)$  with the boundary condition  $\alpha_{\text{TC}}(\Lambda_3) = \alpha_{\text{TC}}(\Lambda_{\text{TC}}) \simeq 0.782 \times \alpha_* \simeq 0.983$  [10] where  $\alpha_*$  is the BZ-IRFP of the one-family  $SU(2)$  TC as the walking/conformal TC,  $\alpha_{SU(3)_2}(\mu) < \alpha_{SU(3)_2}(\Lambda_C) = 0.092 - 0.094$  corresponding to the allowed value of  $1.6 < \kappa_3 < 2.2$ , and  $\alpha_{Y2}(\mu) = \mathcal{O}(10^{-2})$ . Therefore  $\alpha_{U,D,E,N}(\mu)$  do not exceed the critical coupling for the region  $\Lambda_C < \mu < \Lambda_3$ :

$$\begin{aligned} \alpha_{U,D,E,N}(\mu) < \alpha_U(\Lambda_C) &\simeq \frac{3}{4}\alpha_{\text{TC}}(\Lambda_C) + \frac{4}{3}\alpha_{SU(3)_2}(\Lambda_C) + \frac{1}{9}\alpha_{Y2}(\Lambda_C) \\ &\simeq 1.007 - 1.010 < \frac{\pi}{3} \simeq 1.047. \end{aligned} \quad (111)$$

and hence the TC + weak topcolor gauge interactions in the region of  $\Lambda_C < \mu < \Lambda_3$  does not trigger the technifermion condensate.

Next, we consider the criticality in the  $\mu < \Lambda_C$ , where ETC/topcolor/extended hypercharge gauge symmetry break down. The four-fermion interactions involving

only the technifermions at  $\Lambda_C$  are given by

$$\begin{aligned}\mathcal{L}^{4F} = & G_{\text{TC}}^{\text{ETC}} \left( (\overline{Q}_L U_R)^2 + (\overline{Q}_L D_R)^2 + (\overline{L}_L E_R)^2 + (\overline{L}_L N_R)^2 \right) \\ & + G_{\text{topC}}^w \left( (\overline{Q}_L U_R)^2 + (\overline{Q}_L D_R)^2 \right) \\ & + G_{Z'}^{wU} (\overline{Q}_L U_R)^2 + G_{Z'}^{wD} (\overline{Q}_L D_R)^2 + G_{Z'}^{wE} (\overline{L}_L E_R)^2, \quad (112)\end{aligned}$$

where  $(F = U, D, E, N)$

$$G_{\text{TC}}^{\text{ETC}} = \frac{2\pi^2}{M_C^2} \cdot g_{\text{TC}}^{\text{ETC}}, \quad G_{\text{topC}}^w = \frac{4\pi}{4M_C^2} \cdot \frac{\alpha_{\text{QCD}}^2}{\kappa_3}, \quad G_{Z'}^{wF} = \frac{4\pi}{2M_{Z'}^2} \cdot \frac{\alpha_Y^2}{\kappa_1} Y_L^F Y_R^F. \quad (113)$$

Now, the dimensionless four-fermion coupling of technifermions  $U, D, E$  are defined as

$$g_U = N_{\text{TC}} N_c \cdot g_{\text{TC}}^{\text{ETC}} + \frac{N_{\text{TC}} N_c}{2} \cdot \frac{\alpha_{\text{QCD}}^2}{\pi \kappa_3} + \frac{N_{\text{TC}}}{9} \cdot \frac{\alpha_Y^2}{\pi \kappa_1}, \quad (114)$$

$$g_D = N_{\text{TC}} N_c \cdot g_{\text{TC}}^{\text{ETC}} + \frac{N_{\text{TC}} N_c}{2} \cdot \frac{\alpha_{\text{QCD}}^2}{\pi \kappa_3} - \frac{N_{\text{TC}}}{18} \cdot \frac{\alpha_Y^2}{\pi \kappa_1}, \quad (115)$$

$$g_E = N_{\text{TC}} \cdot g_{\text{TC}}^{\text{ETC}} + \frac{N_{\text{TC}}}{2} \cdot \frac{\alpha_Y^2}{\pi \kappa_1}, \quad (116)$$

where  $g_{\text{TC}}^{\text{ETC}} \simeq g_3^{\text{ETC}}$  up to  $\mathcal{O}(1)$  coefficient and Eq.(104) with  $\Lambda_3 = 360 \text{ TeV}$  shows  $N_c g_3^{\text{ETC}} \simeq 10^{-5}$ . The allowed region in Fig. 2 shows  $N_c \alpha_{\text{QCD}}^2 / (\pi \kappa_3) \simeq 10^{-2}$ ,  $\alpha_Y^2 / (\pi \kappa_1) \simeq 10^{-2}$ . The four-fermion couplings are negligible compared with the gauge dynamics, TC + SM gauge interactions, whose running effects yield  $\alpha_{U,D}(\mu) > \pi/3$  at  $\mu \simeq 470 \text{ GeV}$  and hence the dynamical masses of techniquarks

$$m_{\text{TC}} \simeq \mathcal{O}(470 \text{ GeV}), \quad (117)$$

which reproduces the weak scale.

Now we explicitly show that the model realizes TC2 scenario. In this model, the TC theory is walking below  $\Lambda_3 = \Lambda_{\text{TC}}$ . This fact shows that the TC theory develops non-zero anomalous dimension  $\gamma_m^{(\text{TC})}$ . Using  $\gamma_m^{(\text{TC})}$ , the technipion ( $\pi_T$ ) decay constant:  $f_{\pi_T}$  (by Pagels-Stokar formula [32]) and technifermion condensation:  $\langle \overline{F} F \rangle_{M_3}$ , ( $F = U, D, E, N$ ) are represented as

$$\begin{aligned}f_{\pi_T}^2 &= \frac{N}{4\pi^2} \int_0^\infty dx x \frac{\Sigma^2(x) - \frac{x}{4} \frac{d}{dx} \Sigma^2(x)}{(x + \Sigma^2(x))^2} \\ &= \frac{N}{8\pi} \left[ \frac{3 - \frac{\gamma_m^{(\text{TC})}}{2}}{(3 - \gamma_m^{(\text{TC})})^2} \frac{1}{\sin(\frac{\pi}{3 - \gamma_m^{(\text{TC})}})} + \frac{2}{\pi} \left( \ln 2 - \frac{1}{2} \right) \right] m_{\text{TC}}^2, \quad (118)\end{aligned}$$

$$\begin{aligned}\langle \overline{F} F \rangle_{M_3} &= -\frac{N}{4\pi^2} \int_0^{M_3^2} dx \frac{x \Sigma(x)}{x + \Sigma(x)^2} \\ &= -\frac{N}{4\pi^2} \left[ \frac{2}{\gamma_m^{(\text{TC})}} \left( \frac{M_3}{m_{\text{TC}}} \right)^{\gamma_m^{(\text{TC})}} + 1 - \ln 2 \right] m_{\text{TC}}^3, \quad (119)\end{aligned}$$

with  $m_{\text{TC}}$  being a dynamical mass of the technifermion  $\Sigma(x = m_{\text{TC}}^2) = m_{\text{TC}}$ , where

$$\Sigma(x) \sim \begin{cases} m_{\text{TC}} \left( \frac{x}{m_{\text{TC}}^2} \right)^{\frac{1}{2}\gamma_m^{(\text{TC})}-1} & \text{for } x > m_{\text{TC}}^2 \\ m_{\text{TC}} & \text{for } x < m_{\text{TC}}^2 \end{cases}. \quad (120)$$

Eq.(117) and Eq.(118) give  $f_{\pi_T}$  as

$$f_{\pi_T} \simeq 115 \text{ GeV} \quad \text{for} \quad m_{\text{TC}} \simeq 470 \text{ GeV}, \quad (121)$$

where we have used that the anomalous dimension of TC:  $\gamma_m^{(\text{TC})}$  is  $\gamma_m^{(\text{TC})} \simeq 1$  because the combined gauge coupling of TC with weak topcolor  $SU(3)_2$  for  $\Lambda_C < \mu < \Lambda_3$  and the combined one of TC with  $SU(3)_{\text{QCD}}$  for  $\mu < \Lambda_C$  are near critical coupling. Eq.(121) implies that in order to reproduce the weak scale  $(246 \text{ GeV})^2 = 4f_{\pi_T}^2 + f_{\pi_t}^2$ , the top-pion ( $\pi_t$ ) decay constant  $f_{\pi_t}$  should be

$$f_{\pi_t} \simeq 87 \text{ GeV}, \quad (122)$$

and hence the top quark mass  $\hat{m}_t$  coming from the top quark condensation should be

$$\hat{m}_t \simeq 167 \text{ GeV}, \quad (123)$$

where we have used the Pagels-Stokar formula

$$f_{\pi_t}^2 = \frac{3}{8\pi^2} \hat{m}_t^2 \ln \frac{M_C^2}{\hat{m}_t^2}, \quad (124)$$

with  $M_C = 5 \text{ TeV}$  for the constant top quark mass function induced by NJL-type topcolor dynamics corresponding to  $\gamma_m \simeq 2$ <sup>8</sup>.

Let us discuss the third generation quark masses. The ETC induced four-fermion interactions responsible for the third generation quark masses are given by  $\mathcal{L}_{\text{3rd-mass}}$ :

$$\begin{aligned} \mathcal{L}_{\text{3rd-mass}} &= G_3^{\text{ETC}} [(\overline{U}_R Q_L)(\overline{q}_{3L} t_R) + (\overline{D}_R Q_L)(\overline{q}_{3L} b_R)] \\ &\quad + (G_{\text{topC}}^s + G_{Z'}^s)(\overline{t}_R q_{3L})(\overline{q}_{3L} t_R) + [\text{h.c.}] \end{aligned} \quad (125)$$

as discussed in Sec. 3. The bottom quark may acquire mass by technifermion condensation through Eq.(125) in the same way as the ordinary ETC model.

$$m_b = G_3^{\text{ETC}} \langle \overline{D}_R Q_L \rangle \quad (126)$$

On the other hand, the top quark may acquire mass by both technifermion condensation through Eq. (125) and top quark condensation,

$$\begin{aligned} m_t &= G_3^{\text{ETC}} \langle \overline{U}_R Q_L \rangle + (G_3^{\text{ETC}} + G_{\text{topC}}^s + G_{Z'}^s) \langle \overline{t}_R q_L \rangle \\ &\equiv m_t^{(0)} + \hat{m}_t, \end{aligned} \quad (127)$$

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<sup>8</sup>Including QCD log correction decreases  $f_{\pi_t}$  by 10% from  $f_{\pi_t}$  in Eq.(124).

under the conditions Eq.(101)–(103).

Eq.(125) gives the bottom quark mass  $m_b(\Lambda_3)$  as

$$\begin{aligned} m_b(\Lambda_3) &= G_3^{\text{ETC}} \langle \bar{D}_R Q_L \rangle_{M_3} \simeq \frac{4\pi}{2M_3^2} \cdot (0.782 \times \alpha_*) \cdot \langle \bar{D}_R Q_L \rangle_{M_3} \\ &\simeq 0.11 \text{ GeV} \times \left( \frac{m_{\text{TC}}}{470 \text{ (GeV)}} \right)^2 \times \left( \frac{360 \text{ (TeV)}}{\Lambda_3} \right), \end{aligned} \quad (128)$$

and ETC induced top quark mass  $\hat{m}_t^{(0)}(\Lambda_3)$  is the same as  $m_b(\Lambda_3)$ .

However, due to the topcolor dynamics of NJL-type, these masses are greatly amplified as follows. In the NJL-type dynamics there arises large anomalous dimension  $\gamma_m \simeq 2$  near the critical coupling even in the symmetric phase [28] and the bare mass at the scale  $\Lambda$  is greatly enhanced at lower energy scale  $\mu (\ll \Lambda)$

$$m_\mu = Z_m^{-1} m_\Lambda \quad , \quad Z_m^{-1}(\Lambda/\mu) = \left( \frac{\Lambda}{\mu} \right)^2 \cdot \left[ \frac{\ln(\Lambda/\Lambda_{\text{QCD}})}{\ln(\mu/\Lambda_{\text{QCD}})} \right]^{-A/2}, \quad (129)$$

where the logarithmic correction to  $\gamma_m = 2$  comes from QCD correction with  $A = 24/(33 - 2N_f) > 1$  and  $A = 8/7 > 1$  for  $N_f = 6$  ( $\mu < m_{\text{TC}}$ ) and  $A = 24/13 > 1$  for  $N_f = 10$  ( $m_{\text{TC}} < \mu < M_C$ ). Note that the gauged NJL model with  $A > 1$  is renormalizable [29, 30]. In the case at hand  $\Lambda = M_C$  and  $\mu = m_t$  and hence  $Z_m^{-1}(M_C/m_t) \simeq 560$ , so we have

$$m_b(m_t) = \hat{m}_t^{(0)}(m_t) \simeq 5 \text{ GeV}, \quad (130)$$

for  $\Lambda_3 \simeq 4500 \text{ TeV}$  and  $M_C = 5 \text{ TeV}$ .

If we arrange the ETC breaking scales  $\Lambda_{1,2}$  somewhat higher than that of the third generation  $\Lambda_3$  so that the combined four-fermion interactions of topcolor, extra  $U(1)$  and ETC are off the criticality, then the mass of the second and the first generation fermions would have no large enhancement due to anomalous dimension and hence give reasonable hierarchy compared with the top and bottom.

Leptons would acquire masses from the technilepton condensate  $\langle \bar{E} E \rangle$  whose gauge coupling  $\alpha_E \simeq 3/4 \times \alpha_{\text{TC}} \simeq 3/4 \times \alpha_* = 3\pi/10$  is smaller than the critical coupling  $\pi/3$  in the ladder approximation. However, as discussed in Sec. 3 there is some ambiguity in evaluation of the critical coupling up to 1-20% in the ladder approximation [36]. In fact the critical coupling  $\alpha^{\text{crit}}$  decreases by 20% when we define the critical coupling such that an anomalous dimension is  $\gamma_m = 1$  at two-loop level [10]. Thus, up to this ambiguity leptons may acquire mass somewhat smaller than the mass of quarks [40].

## 6 Top-pion Mass

We here discuss a novel effect of the large anomalous dimension of the topcolor on the estimation of the top-pion mass  $m_{\pi_t} < 70 \text{ GeV}$ , which is extremely smaller than the

conventional estimation  $\simeq 200 - 300 \text{ GeV}$  [9]. This applies to generic TC2 model not restricted to ours.

The top-pion appears in the generic TC2 model[25, 26], since both the top quark condensate and technifermion condensate break respective global symmetries, which results in two kinds of three Nambu-Goldstone (NG) bosons. Three of them (mainly boundstates of technifermions) are absorbed into  $W, Z$  bosons as usual. The rest three are pseudo NG bosons (mainly boundstates of top and bottom), which is called the top-pion. In general, the former three NG bosons decay constant  $f_{\pi_T}$  and the top-pion decay constant  $f_{\pi_t}$  reproduce the weak scale  $(246 \text{ GeV})^2 = N_D f_{\pi_T}^2 + f_{\pi_t}^2$ , where  $N_D$  is the number of the technifermion EW-doublet.

The top-pion mass may be estimated by the Dashen formula [31]

$$m_{\pi_t}^2 f_{\pi_t}^2 = -m_t^{(0)} \langle \bar{t}t \rangle, \quad (131)$$

where  $m_t^{(0)}$  is the ETC-induced top quark mass acting as the bare mass for the NJL-type topcolor dynamics.

Now, we recall that the renormalization of generic mass parameter  $m$  is carried out with keeping such a relation as

$$m_\Lambda (\bar{\psi}\psi)_\Lambda = m_\mu (\bar{\psi}\psi)_\mu, \quad \Lambda \gg \mu, \quad (132)$$

where  $\psi$  is the generic fermion and the suffix  $\Lambda(\mu)$  represents a bare (renormalized) quantity at  $\Lambda(\mu)$  ( $\mu$  is the reference renormalization point). Eq.(132) shows that when we write the renormalization of the mass parameter  $m$  as

$$m_\Lambda = Z_m m_\mu, \quad (133)$$

then the renormalization of the composite operator  $(\bar{\psi}\psi)$  should be given by

$$(\bar{\psi}\psi)_\Lambda = Z_m^{-1} (\bar{\psi}\psi)_\mu, \quad (134)$$

where  $Z_m^{-1} = (\Lambda/\mu)^{\gamma_m}$  is the renormalization factor and  $\gamma_m$  is the anomalous dimension of the mass parameter. In the TC2 model, the condensation  $\langle \bar{t}t \rangle$  at  $M_C$  is represented by [25]

$$\langle \bar{t}t \rangle|_{M_C} = -\frac{N_c}{4\pi^2} \int_0^{M_C^2} dx \frac{x \Sigma(x)}{x + \Sigma(x)^2} = -\frac{N_c}{4\pi^2} \cdot \hat{m}_t M_C^2, \quad (135)$$

for the constant top quark mass  $\hat{m}_t (= m_t - m_t^{(0)})$  induced by the NJL-type topcolor dynamics corresponding  $\gamma_m \simeq 2$ . Eq.(134) relates  $\langle \bar{t}t \rangle|_{M_C}$  to  $\langle \bar{t}t \rangle|_{\hat{m}_t}$  as

$$\langle \bar{t}t \rangle|_{M_C} = Z_m^{-1} \langle \bar{t}t \rangle|_{\hat{m}_t} = \left( \frac{M_C}{\hat{m}_t} \right)^2 \langle \bar{t}t \rangle|_{\hat{m}_t}, \quad (136)$$

and the condensation  $\langle \bar{t}t \rangle|_{\hat{m}_t}$  is represented by

$$\langle \bar{t}t \rangle|_{m_t} = -\frac{N_c}{4\pi^2} \int_0^{\hat{m}_t^2} dx \frac{x\Sigma(x)}{x + \Sigma(x)^2} = -\frac{N_c}{4\pi^2} \cdot \hat{m}_t^3. \quad (137)$$

On the other hand, Eq.(133) relates the ETC-induced top quark mass  $\hat{m}_t^{(0)}(M_C)$  to  $\hat{m}_t^{(0)}(\hat{m}_t)$  as

$$m_t^{(0)}(M_C) = Z_m m_t^{(0)}(\hat{m}_t) = \left( \frac{\hat{m}_t}{M_C} \right)^2 m_t^{(0)}(\hat{m}_t). \quad (138)$$

Therefore, we observe that the right-hand side of Eq.(131) is *renormalization point independent*:

$$m_t^{(0)}(M_C) \cdot \langle \bar{t}t \rangle|_{M_C} = m_t^{(0)}(\hat{m}_t) \cdot \langle \bar{t}t \rangle|_{\hat{m}_t}, \quad (139)$$

which shows that in the TC2 model we have the generic form as

$$m_{\pi_t}^2 f_{\pi_t}^2 = m_t^{(0)}(\hat{m}_t) \cdot \frac{N_c}{4\pi^2} \hat{m}_t^3. \quad (140)$$

Now, the top-pion decay constant  $f_{\pi_t}$  is evaluated in exactly the same way as in the original top quark condensate paper [21]. The Pagels-Stokar Formula gives  $f_{\pi_t}$  as a function of  $\hat{m}_t$  (top quark mass coming from the top quark condensate) and  $M_C$  :

$$f_{\pi_t}^2 = \frac{3}{8\pi^2} \hat{m}_t^2 \ln \frac{M_C^2}{\hat{m}_t^2}, \quad (141)$$

for the constant top quark mass induced by the NJL-type topcolor dynamics corresponding  $\gamma_m \simeq 2$ .

Thus Eq.(140) and (141) give us the generic form of the top-pion mass as

$$m_{\pi_t}^2 = 2 \cdot m_t^{(0)}(\hat{m}_t) \cdot \frac{\hat{m}_t^3}{\hat{m}_t^2 \cdot \ln(M_C^2/\hat{m}_t^2)} = \frac{m_t^{(0)}(\hat{m}_t) \cdot \hat{m}_t}{\ln(M_C/\hat{m}_t)}. \quad (142)$$

Experimentally, a model-independent lower limit [33] of the coloron mass  $M_C$  is

$$\begin{aligned} M_C/\cot\theta &> 837 \text{ GeV (flavor - universal)}, \\ M_C/\cot\theta &> 450 \text{ GeV (flavor - non - universal)}, \end{aligned} \quad (143)$$

for the flavor-universal coloron and the flavor-non-universal coloron, respectively, where  $\cot\theta = h_1/h_2 > 1$ , because  $h_{1(2)}$  represents the gauge coupling of the strong (weak) topcolor  $SU(3)_{1(2)}$ .

Hence Eq.(142) and (143) yield an upper bound of top-pion mass

$$m_{\pi_t}^2 < \frac{m_t^{(0)}(\hat{m}_t) \cdot \hat{m}_t}{\ln(M_C/\hat{m}_t)} = \frac{m_t^{(0)}(\hat{m}_t) \cdot [m_t - m_t^{(0)}(\hat{m}_t)]}{\ln(M_C/\hat{m}_t)}, \quad (144)$$

where physical top quark mass is  $m_t = \hat{m}_t + m_t^{(0)} = 172 \text{ GeV}$ . Eq.(144) has the maximum value at  $m_t^{(0)}(\hat{m}_t) \simeq 70 \text{ GeV}$  as

$$\begin{aligned} m_{\pi_t}^2 &< \frac{m_t^{(0)}(\hat{m}_t) \cdot \hat{m}_t}{\ln(M_C/\hat{m}_t)} \simeq (60 \text{ GeV})^2, \\ &\simeq (70 \text{ GeV})^2, \end{aligned} \quad (145)$$

for the flavor-universal coloron case and the flavor non-universal case, respectively. Eq.(145) is a very conservative upper bound universal to generic model of TC2 not restricted to specific TC2 model, since in the generic TC2 model we have actually  $\cot \theta = h_1/h_2 > 4$  instead of  $\cot \theta = h_1/h_2 > 1$  in order to trigger the top quark condensate (see Eq.(79)) [9].

Our estimate above is quite different from the conventional estimate which is made at the scale  $M_C$ :  $m_t^{(0)}(M_C) \cdot \langle \bar{t}t \rangle|_{M_C}$ , where  $\langle \bar{t}t \rangle|_{M_C} = [N_c(\hat{m}_t)^3/(4\pi^2)] \cdot (M_C/\hat{m}_t)^2$  up to logarithm.  $\langle \bar{t}t \rangle|_{M_C}$  is larger than that estimated at  $\hat{m}_t$  by  $Z_m^{-1} \simeq (M_C/\hat{m}_t)^2$  as it should. The crucial point is the evaluation of  $m_t^{(0)}(M_C)$ , which is usually made independently of  $m_t^{(0)}(\hat{m}_t)$  and taken as on the order of  $m_b \simeq 5 \text{ GeV}$ . However, if we do this, the physical mass of the bottom  $m_b(\hat{m}_t)$  and the ETC-driven mass of the top  $m_t^{(0)}(\hat{m}_t)$  should be enhanced by the same factor  $Z_m^{-1} \simeq (M_C/\hat{m}_t)^2$  into absurdly large value. Or,  $m_t^{(0)}(M_C)$  must be taken to be  $Z_m$  times smaller than the physical value  $m_t^{(0)}(\hat{m}_t)$  which should be a small portion of the top quark mass in the TC2 scenario  $m_t^{(0)}(\hat{m}_t) \ll m_t \simeq 172 \text{ GeV}$ . This is the effect of the large anomalous dimension. Anyway the result should be the same as ours as far as we correctly take account of *renormalization-point independence* of the operator  $m_t^{(0)}\bar{t}t$  which is multiplicatively renormalized, since the gauged NJL model in this case is renormalizable (See the previous footnote 4).

## 7 Summary and discussions

We have experimented a straightforward explicit ETC model building which incorporates the top quark condensate via universal coloron type topcolor  $SU(3)_1 \times SU(3)_2$  which is spontaneously broken to the ordinary  $SU(3)_{\text{QCD}}$ . All the quarks and techniquarks were assigned to have only  $SU(3)_1$  which is much stronger than  $SU(3)_2$  to trigger the top quark condensate.

The criticality conditions of MAC for the  $SU(5)$  ETC and  $SU(2)$  hypercolor dynamics realized the successive ETC breakings down to  $SU(2)$  TC which is walking/conformal near the conformal window. Imposing the criticality conditions at each step of ETC breaking predicted the ETC breaking scales somewhat larger than those of Ref. [18, 19] and hence very small ETC-driven masses of the third generation quarks/leptons, of order  $\mathcal{O}(10^{-1} \text{ GeV})$ , in spite of the enhancement of large anomalous dimension  $\gamma_m \simeq 1$  of the walking/conformal TC.

Imposing the topcolor  $SU(3)_1$  to be near the criticality, we realized the top quark condensate so as to give a realistic mass to the top quark. However, the techniquarks

then feel both the strong topcolor near criticality as well as the equally strong walking/conformal TC near the criticality. Such combined strong gauge interactions trigger the techniquark condensate at the scale ridiculously large compared with the weak scale.

Then we considered an alternative model of TC2, with coloron and  $Z'$  of flavor-universal type, where the quarks have strong  $SU(3)_1 \times U(1)_{Y_1}$  interactions, while techniquarks do weak  $SU(3)_2 \times U(1)_{Y_2}$  interactions, with both spontaneously broken to the SM gauge theories  $SU(3)_{\text{QCD}} \times U(1)_Y$ . Since explicit ETC model of this charge assignment is rather involved, we only considered here the effective theory of TC2 assuming that similar ETC breaking can take place in a larger picture of a certain ETC gauge group. In such a framework we discuss the mass of third generation quarks can be realistic even when all the ETC breaking scales are somewhat larger than usually considered as we demonstrated in the explicit ETC model. A key observation was that the ETC-driven mass of quarks are regarded as the bare mass of the topcolor sector, which then can be enormously enhanced by the large anomalous dimension  $\gamma_m \simeq 2$  of the NJL-type dynamics of the broken topcolor, if the effective four-fermion coupling is near the criticality:  $m_t^{(0)}(m_t) = Z_m^{-1} m_t^{(0)}(M_C)$  with  $Z_m^{-1} \simeq (M_C/m_t)^{\gamma_m} \simeq (M_C/m_t)^2$ , which is typically  $Z_m^{-1} \simeq 500$  for the coloron mass  $M_C > 4 \text{ TeV}$ . We then obtained realistic masses  $m_b \simeq 5 \text{ GeV}$  as well as  $m_t \simeq 172 \text{ GeV}$  whose main part  $\simeq 167 \text{ GeV}$  comes from the top quark condensate and the rest  $\simeq 5 \text{ GeV} (\simeq m_b)$  is the ETC origin mass enhanced by the anomalous dimension  $\gamma_m \simeq 2$ . If we arrange the ETC breaking scales somewhat higher than that of the third generation so that the combined four-fermion interactions of topcolor, extra  $U(1)$  and ETC are off the criticality, then the mass of the second and the first generation fermions would have no large enhancement due to anomalous dimension and hence give reasonable hierarchy compared with the top and bottom.

Another possibility to modify the top-mode ETC type model would be to put the fourth generation in stead of the technifermion and the SM fermions into the same representation of a horizontal group, say  $SU(4)$ , in such a way that the fourth quarks have the same strong flavor-universal topcolor  $SU(3)_1$  as that of the three generations quarks. Then the fourth generation quark condensate triggered by the topcolor would play the role of the technifermion condensate. In order that only the fourth quark ( $t'$ ,  $b'$ ) and the top quark should condense, we should arrange the ETC-type interactions to discriminate them from others in such a way that the effective four-fermion couplings are arranged as  $g_{t'} > g_{t'}^{\text{crit}}$ ,  $g_{b'} > g_{b'}^{\text{crit}}$ ,  $g_t > g_t^{\text{crit}}$ , where  $g_i^{\text{crit}}$  ( $i = t', b', t$ ) is the critical line of the gauged NJL model having the SM gauge interaction contributions (see Sec. 4). Of course, the fourth generation neutrino should have Majorana condensate in order to avoid the light fourth neutrino. Explicit ETC-type model having successive symmetry breaking of the horizontal symmetry would be interesting.

Finally, we found a novel effect of the large anomalous dimension  $\gamma_m \simeq 2$  of the NJL-type dynamics on the evaluation of the top-pion mass through the Dashen formula together with the Pagels-Stokar formula. The Dashen formula contains the bare mass



(ETC-induced mass) times top quark condensate. Usual estimate of this combination is made at the scale of the topcolor breaking (coloron mass)  $M_C$ : The condensate has an enhancement of quadratic divergence of NJL-type  $\sim M_C^2$ , while the bare mass at  $M_C$  scale was just assumed to be the order of the physical bottom mass  $\sim m_b$ , which is, however, enormously enhanced as much as  $10^2 - 10^3$  times by the renormalization effect  $Z_m^{-1} \simeq (M_C/m_t)^2$  due to the same quadratic divergence when evaluated at the scale of  $m_t$ . Based on the renormalization invariance of the product of the bare mass and the condensate, we estimated it at the scale of physical  $m_t$ . Our most conservative estimate turned out to be very small  $m_{\pi_t} < 70 \text{ GeV}$  which is universal to generic TC2 model. This would give a serious impact on the phenomenology of the generic TC2 model and similar models having two kinds of NG bosons, one linear combination of which is absorbed into  $W, Z$  bosons and the rest remaining as pseudo NG bosons, particularly when they are produced by the NJL-type dynamics.

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